Electrical Machines-2

Digital Notes

Prepared by

Prof. K Subhas

Department of EEE

MRCET

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UNIT-I

Single Phase Transformers

- **Principle of operation**
- **Constructional details and types**
- **E.M.F equation**
- **Equivalent circuit**
- **Operation on no load and on load**
- **Phasor diagrams**
- **Losses**
- **Minimization of hysteresis and eddy current losses**
- **Efficiency-all day efficiency-regulation**
- **Effect of variations of frequency and supply voltage on iron losses**

Introduction:

A transformer is a device used to transfer Electric power from one circuit at a certain voltage level to another circuit at a different voltage level. The electrical power is transferred by Magnetic Induction between two coils in the magnetic circuit of the transformer. It is an electrical equipment which has the highest efficiency since there are no moving parts . Transformers can carry only AC Electrical power. Transformers are available in single phase and three phase but we will study only single phase transformers in this unit.

Types and applications of transformers:

1. Step up/Unit transformers – Usually located at the output of a generator in Generating stations . Its function is to step up the voltage level to transmit power with minimum losses.
2. Step down/Substation transformers – Located at main distribution or secondary level transmission substations. Its function is to lower the voltage levels for first level distribution
3. Distribution Transformers – located at small distribution substation. It lowers the voltage levels for second level distribution purposes.
4. Special Purpose Transformers - E.g. Potential Transformer (PT) , Current Transformer (CT) etc which are used for metering applications.

Principle of operation of Transformers:

In a generator a voltage is induced in a coil which moves past a stationary magnetic field emanating from field coils and the field flux is constant. But since the flux that links with the coil is changing, a voltage is induced in the coil which is proportional to the rate of change of flux linkage. In a transformer though both the coils and the magnetic circuit are stationary, a voltage is induced in the secondary since the current which flows in the primary is alternating and it produces a continuously changing flux. According to Faraday's laws of electromagnetic Induction when the magnetic flux linking with a conductor (or Coil) changes, an EMF is induced in that.

In a single phase transformer there are two coils one known as primary and the other known as secondary both wound on he same magnetic circuit (core)of high permeability (or low reluctance) as shown in the figure below.

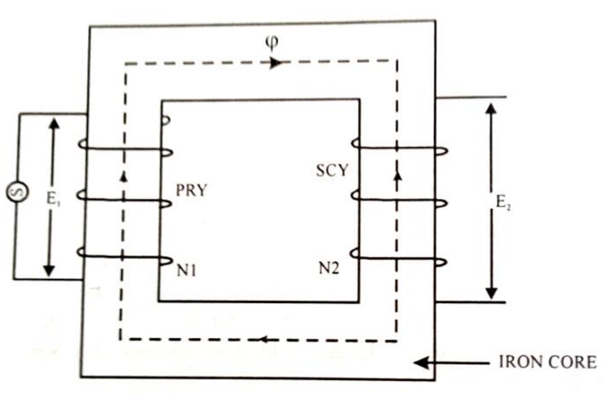


Fig: Single Phase Transformer

An alternating Voltage is applied to the primary coil sets up an alternating current in the primary coil and hence an alternating flux in the magnetic circuit (Core).This magnetic flux links with the secondary coil and induces in it an alternating voltage due to the changing flux linkage. The EMF induced in the secondary coil will be supplied to the load.

Ideal Transformer:
An ideal transformer has no losses i.e. no I^2R losses (its windings have no resistance) and no core losses. In other words it consists of two pure inductive coils and a loss less core. But practically it is not possible to realize such an Ideal transformer.
In an Ideal transformer the no load current will be lagging the applied voltage by 90^0 since the current flows through a purely inductive primary coil. The flux **Ø** induced in the magnetic circuit also will be lagging the applied voltage V_1 since the current in the primary is entirely a magnetizing current (the coil is assumed to be totally inductive) and will be in phase with the magnetic flux. The voltages induced in the primary and secondary coils E_1 **&** E_2 will be lagging the flux by 90^0 and in phase opposition to the applied Voltage V_1. All these relations are clearly shown in the No load phasor diagram given below.

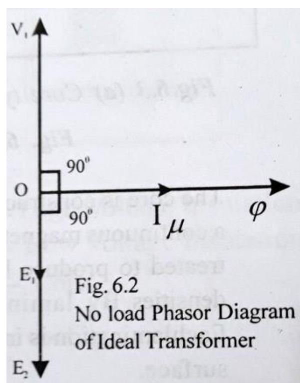


Fig: No load Phasor Diagram of an Ideal Single phase Transformer

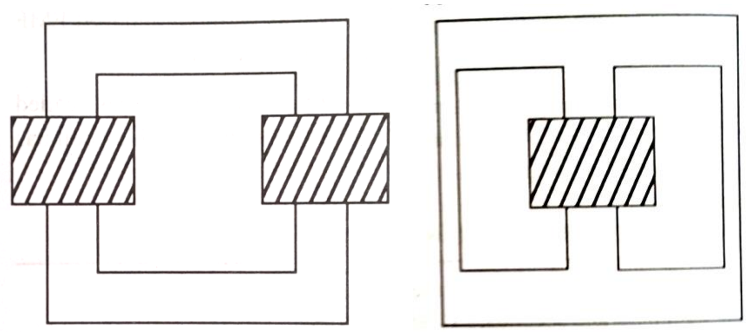
Types and constructional features:

The essential parts of a Transformer are :

- Coils and the Laminated core
- Tank or container to house the coils & the core
- Suitable medium called the transformer oil to insulate the core and the windings from the tank
- Bushings (porcelain) for insulating and bringing out the coil terminals.

There are two types of core construction called ***core*** type and ***shell*** type and are shown in the figure below.

- ***In Core Form*** the transformer windings surround the two sides of a rectangular core.
- ***In Shell Form*** a three legged laminated core is used with the windings wrapped around the centre leg.



(a) (b)

Fig: (a) Core type and (b) Shell type transformers

The core is constructed of laminations to minimize the ***eddy current*** losses. The laminations are made of thin sheet steel with high silicon content to produce *high permeability* and a *low hysteresis loss.* The laminations are insulated from each other by a coat of varnish. The laminations are cut in the form of strips of shape L, E & I called *core stampings* as shown in the figure below and then stacked together to get the *Core* and *shell* type cores.

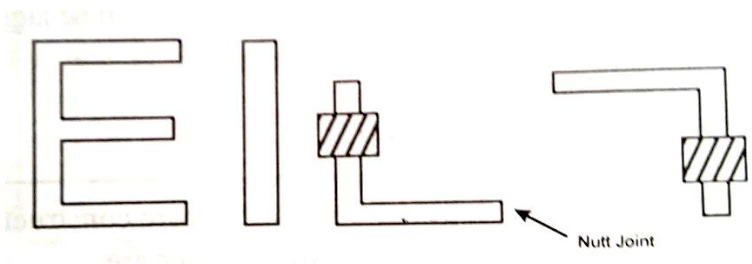


Fig: Types of core stampings

EMF Equation of a Transformer:

- Number of turns in the primary and secondary windings : N_1 **and** N_2
- Maximum value of flux in the core : $Ø_m$
- Frequency : **f**
- Time for change of Flux in one cycle : **1/f**
- Time for change of flux from zero to $Ø_m$: **1/4f**
- Hence rate of change of flux : $Ø_m/1/4f$
- Average e.m.f. induced in each turn : $4f\,Ø_m$
- Form factor for a sine wave : **1.11**
- RMS value of voltage in a turn : $1.11 \times 4f\,Ø_m = 4.44\,f\,Ø_m$
- RMS value of voltage induced in the primary : $4.44\,f\,Ø_m N_1$
- RMS value of voltage induced in the secondary : $4.44\,f\,Ø_m N_2$

It is to be noted that when a sinusoidal voltage V_1 is applied to the primary winding leading to the flow of current and thus generation of flux in the core, voltages are induced in both the primary and secondary windings as given by the equations above. Voltage E_1 **&** E_2 are opposite in polarity to V_1 and the voltage E_2 gives rise to the load current when the secondary is connected to a load.

We get the relation between the two voltages by dividing the expressions for E_1 with that of E_2 as

$$E_1/E_2 = N_1/N_2$$

And this ratio is called the ***turns ratio*** **'a'** or **'k'** of the transformer.

Since the primary ***VoltAmp*** **(VA)** is nearly equal to that of the secondary due to the high efficiency of transformers we get the relation **:** $E_1 I_1 = E_2 I_2$ **i.e** $I_1/I_2 = E_2/E_1 = N_2/N_1 = 1/a$ **or** $1/k$

A practical Transformer on no load:

When the primary winding of a transformer is energized with an AC source of power and the secondary is left open without connecting to any load ,the Transformer is said to be on No load.

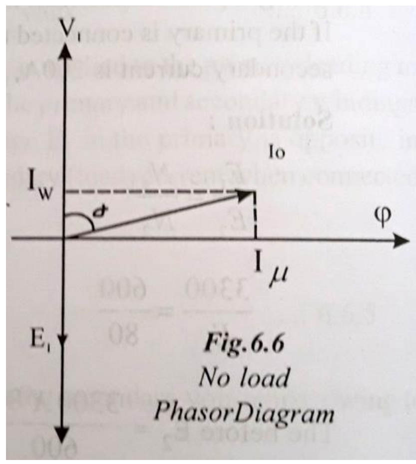


Fig: Phasor diagram of a practical single phase transformer on No Load

In a practical Transformer the primary winding cannot be a pure reactance and it would have some resistance also. Then the primary no-load current I_0 would lag behind the applied voltage by an angle α which is slightly less than 90^0 and it can be resolved into the following two components as shown in the Phasor diagram .

- $I_\mu = I_0 \sin \alpha$ along the direction of flux called the magnetizing current and is responsible for the generation of flux **Ø** in the core of the transformer
- $I_w = I_0 \cos \alpha$ along the direction of the primary voltage V_1 called the working (active)component and is responsible to cover the no load losses (*Hysteresis, eddy current and small resistive*)

In this diagram the applied voltage V_1 and the flux **Ø** will have phase difference of 90^0 and are taken as reference.

The no load losses are given by: $P_{NL} = V_1 . I_W$

E_1 and E_2 are the induced voltages in the primary and secondary and both lag behind the flux by 90^0 since the induced voltage is equal to the rate of change of flux linkages.

Equivalent circuit of a real transformer:

The equivalent circuit of a transformer is a circuit which will take into account all the major imperfections in a practical transformer and are modeled appropriately as equivalent resistors and inductors as explained below.

To develop the equivalent circuit of a real transformer, the following losses have to be taken into account in order to accurately model the transformer into its equivalent circuit:

- *Copper (I^2R) Losses* – Resistive heating losses in the primary and secondary windings of the transformer.

They are modeled by placing a resistor R_P in the primary circuit and a resistor R_S in the secondary circuit.

- *Eddy current Losses* – Resistive heating losses in the core of the transformer.
- *Hysteresis Losses* – These are associated with the rearrangement of the magnetic domains in the core during each half-cycle.

Both theses losses produce a core loss current I_{H+E} or I_{CL} which is a current proportional to the voltage applied to the core. Since this is in phase with the applied voltage this loss is modeled as a resistance R_C across the primary voltage source.

- Leakage flux – The fluxes $Ø_{LP}$ and $Ø_{LS}$ which escape the core and pass through only one of the transformer windings are called leakage fluxes. They then produce self-inductances in the primary and secondary coils.

They are modeled as equivalent Inductive reactances X_P and X_S in the primary and secondary circuits in series with the resistors R_P and R_S.

Apart from the above losses the transformer draws a magnetizing current I_μ and since this current lags behind the applied voltage by 90^0 it is modeled as a reactance X_M

Complete Equivalent circuit of a Practical Transformer incorporating all the above aspects is shown in the figure below.

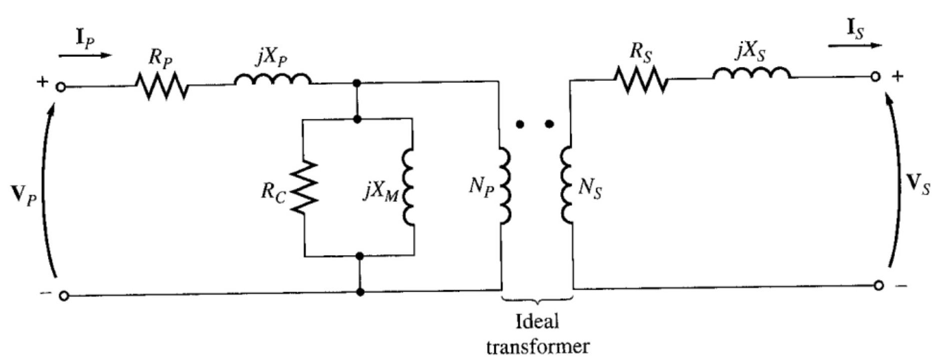


Fig: Complete Equivalent circuit of a Practical Transformer

For ease of circuit analysis and mathematical calculation this complete equivalent circuit is simplified by referring the impedances in the secondary to the primary and vice versa and shown below.

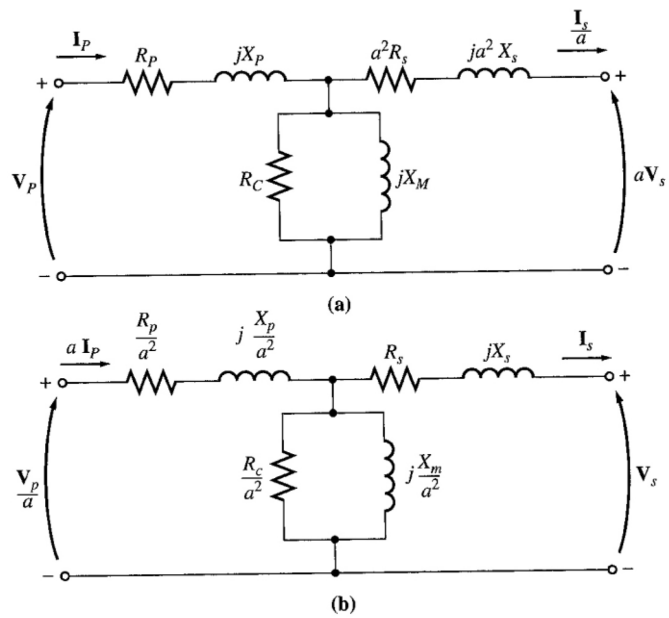


Fig: Equivalent circuit of a Transformer (a) referring to the primary (b) referring to the secondary

- When the ***Secondary*** is reflected to the *Primary* the Secondary side parameters reflected to the Primary will be come: **$a.V_S$, I_S/a , $a^2. R_S$ and $a^2 X_S$**
- When the ***Primary*** is reflected to the *Secondary* the Primary side parameters reflected to the Secondary will be come: **V_p/a , $a.I_p$, R_p / a^2 and X_p/ a^2**

Approximate Equivalent circuit of a Transformer:

The derived equivalent circuit is detailed but it is complex for practical engineering applications. The main problem in analysis & calculations is representation of the excitation and the eddy current & hysteresis losses which add an extra branch.

In practical situations, the excitation current I_μ & the core loss current I_{CL} will be relatively small as compared to the load current, which makes the resultant voltage drop across R_p and X_p to be very small. Hence R_p and X_p may be lumped together with the secondary referred impedances to form an equivalent impedance. In some cases, the excitation current I_μ & the core loss current I_{CL} are neglected entirely due to their small magnitude.

The equivalent circuits with these simplifications is shown in the figure below.

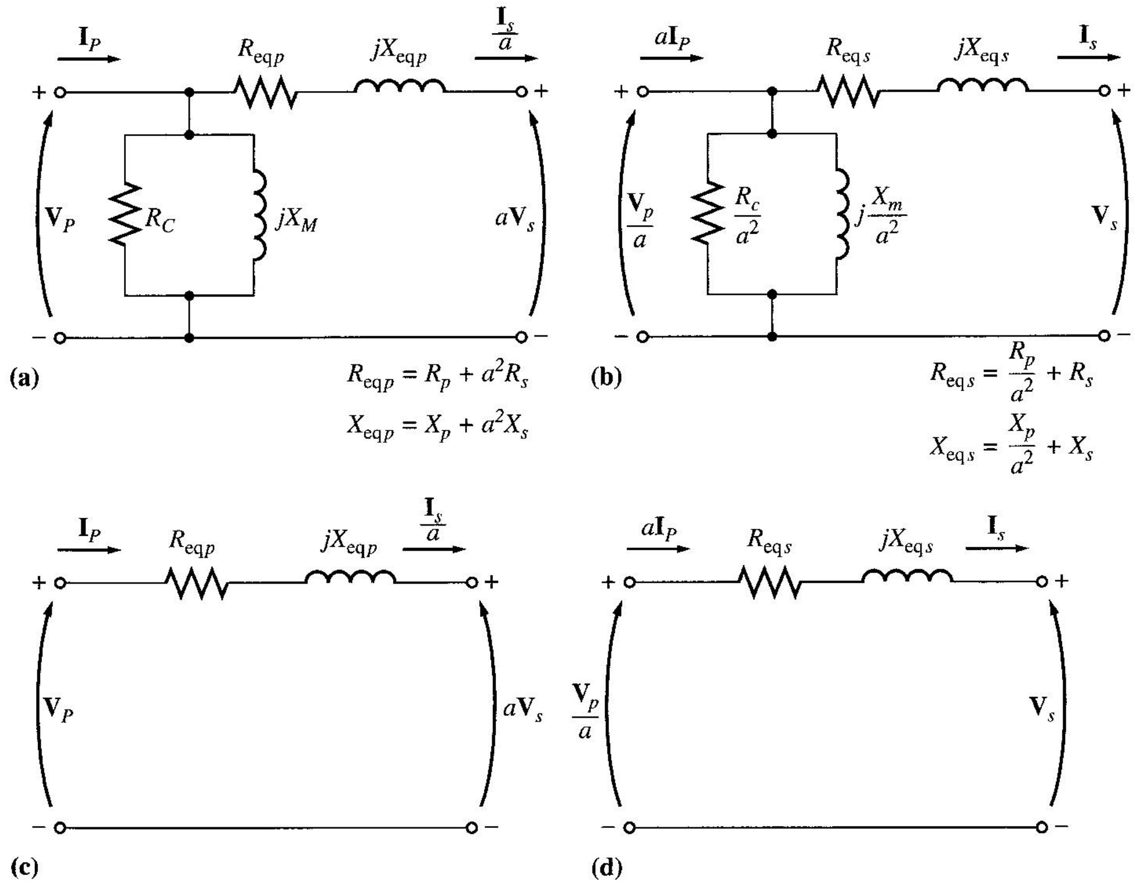


Fig: Simplified equivalent circuits (a) Referred to the primary side (b) Referred to the secondary side (c) With no excitation branch, referred to the primary side (d) With no excitation branch, referred to the secondary side

Voltage Regulation of a Transformer:

The output voltage of a transformer varies with the load even if the input voltage remains constant. This is because of the voltage drop in it's series impedance as seen in the equivalent circuit. It is normally defined for Full load as a percentage as given below:

Voltage Regulation **(VR)** $= [(V_{S\,NL} - V_{S\,FL}) / V_{S\,NL}] \cdot 100\ \%$

The transformer phasor diagram:

To determine the voltage regulation of a transformer, we must understand the voltage drops within the transformer and they can be seen easily with the help of the Phasor diagram **ON LOAD** .

To develop the Phasor diagram on load let us consider once again the simplified equivalent circuit referred to the secondary side:

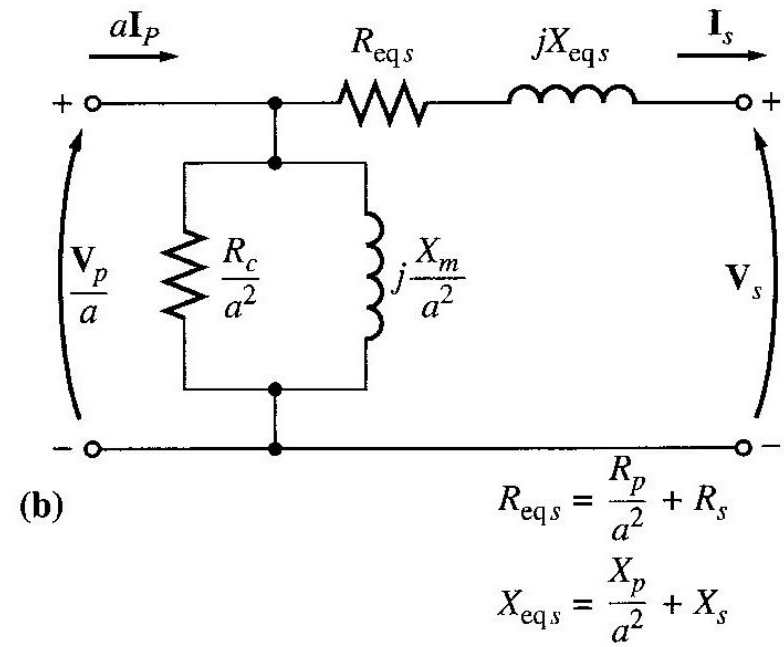


Fig: Simplified equivalent circuit Referred to the secondary side

Ignoring the excitation branch (since the current flow through the branch is considered to be small), Series impedances **(R_{eq} + jX_{eq})** will be more predominant. Hence Voltage Regulation depends on the magnitude of the series impedance and the phase angle of the load current flowing through the secondary of the transformer with respect to the secondary voltage.

Phasor diagram will represent the effects of these factors on the voltage regulation. A phasor diagram consists of current and voltage vectors. In this Phasor diagram the secondary voltage V_S is assumed as the reference.

Applying Kirchoff Voltage Law to this simplified equivalent circuit we get the relation :

$$V_P/a = V_S + I_S \cdot R_{eqs} + I_S \cdot X_{eqs}$$

From this equation, the phasor diagram can be visualized and is shown below for a transformer operating at a lagging power factor. For lagging loads, $V_P / a > V_S$ so the voltage regulation with lagging loads is **> 0**.

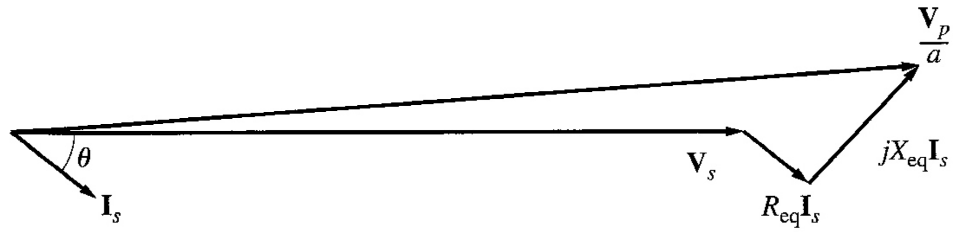


Fig: Phasor diagram of a Transformer on load with lagging power factor

When the power factor is unity also V_S is lower than V_P and so **VR > 0**. But **VR** is smaller than before (during lagging PF) as shown in figure (a) below .

With a leading power factor V_S is higher than the referred V_P so **VR < 0** as shown in figure (b) below.

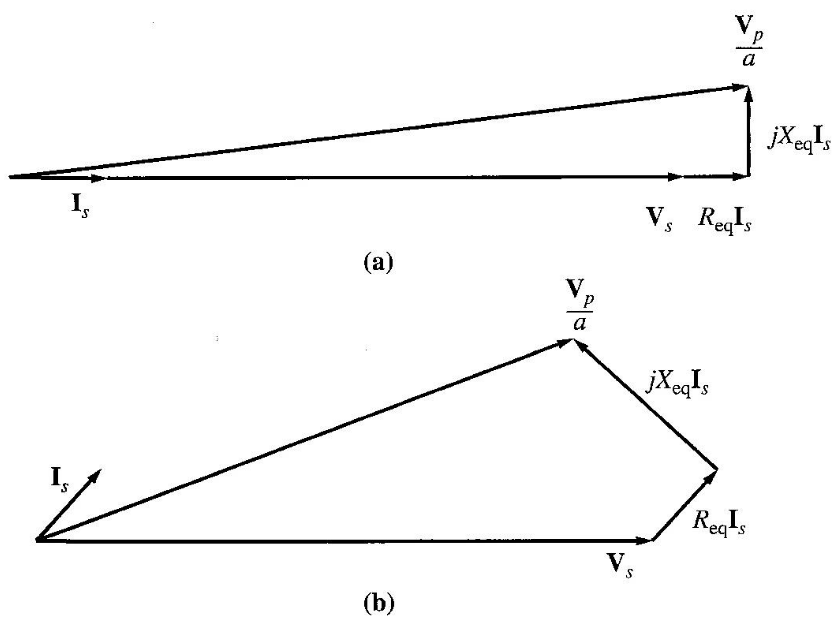


Fig: Phasor diagram of a Transformer on load with (a) unity power factor and (b) leading power factor

Efficiency of a Transformer:

Efficiency of a Transformer is defined as (Same as defined for motors and generators):

$$\textbf{Efficiency}\quad \eta = (P_{out} / P_{in}).100\% = (P_{out} / P_{out} + \text{Losses}).100\%$$

Losses incurred in a transformer:

- Copper losses **(I^2R)**

- Core losses (Hysteresis losses and Eddy current losses)

Therefore, for a transformer, efficiency may be calculated using the following:
From the phasor diagram we know that P_{out} in a transformer is given by $P_{out} = V_S.I_S.\ Cos\ \Theta$. Hence

$$\eta = (V_S.I_S.\ Cos\ \Theta \,/\, V_S.I_S.\ Cos\ \Theta + P_{CU} + P_{CORE}).100\%$$

All Day Efficiency of a Transformer

Definition: All day efficiency means the power consumed by the transformer throughout the day. It is defined as the ratio of output power to the input power in kWh or wh of the transformer over 24 hours. Mathematically, it is represented as

$$\text{All day efficiency, } \eta_{\text{all day}} = \frac{\text{output in kWh}}{\text{input in kWh}} \quad \text{(for 24 hours)}$$

All day efficiency of the transformer depends on their load cycle. The load cycle of the transformer means the repetitions of load on it for a specific period.

The ordinary or commercial efficiency of a transformer define as the ratio of the output power to the input power.

$$\eta = \frac{\text{output power}}{\text{input power}} = \frac{\text{output power}}{\text{output power} + \text{losses}}$$

What is the need of All Day Efficiency?

Some transformer efficiency cannot be judged by simple commercial efficiency as the load on certain transformer fluctuate throughout the day. For example, the distribution transformers are energised for 24 hours, but they deliver very light loads for the major portion of the day, and they do not supply rated or full load, and most of the time the distribution transformer has 50 to 75% load on it.

As we know, there are various losses in the transformer such as iron and copper loss. The iron loss takes place in the core of the transformer. Thus, the iron or core loss occurs for the whole day in the distribution transformer.The second type of loss known as copper loss takes place in the windings of the transformer also known as the variable loss. It occurs only when the transformers are in the loaded condition.

Hence, the performance of such transformers cannot be judged by the commercial or ordinary efficiency, but the efficiency is calculated or judged by All Day Efficiency also known as operational efficiency or energy efficiency which is computed by energy consumed during 24 hours.

UNIT-II

Testing of Single Phase Transformer and Autotransformer

- **OC and SC tests**
- **Sumpner's test**
- **Predetermination of efficiency and regulation**
- **Separation of losses test**
- **Parallel operation with equal and unequal voltage ratios**
- **Auto transformers**
- **Equivalent circuit**
- **Comparison with two winding transformers.**

Open Circuit (OC) and Short Circuit (SC) tests:

They are conducted on the transformer to find out the transformer losses and from them to determine the circuit constants that are used to represent the equivalent circuit. From these parameters the transformer efficiency and regulation can also be calculated. These tests are conducted without actually loading the transformer to its full load and hence the power consumed during the test is very small as compared to its full load (rated) power.

Open Circuit or No Load test:

The test setup to conduct the OC test is shown in the figure below.

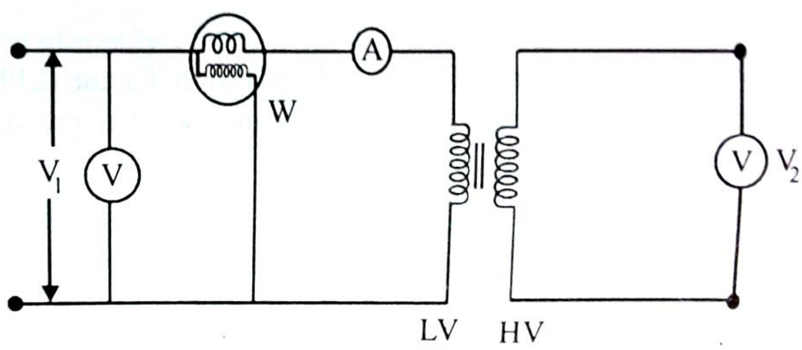


Fig: Test Setup to conduct the Open Circuit or No Load test

Low voltage side is designated as Primary and High voltage side is designated as Secondary. Voltmeter $\mathbf{V_1}$,Ammeter **A** and wattmeter **W** are connected in the primary as shown . Voltmeter $\mathbf{V_2}$ is connected in the open circuited Secondary. Since the secondary is open circuited a small value of no load current called $\mathbf{I_0}$ flows in the primary and this is measured by the ammeter **A.** The power loss in the transformer is due to core losses and a very small $\mathbf{I^2R}$ loss in the primary. There is no $\mathbf{I^2R}$ **loss** in the secondary since the secondary is open and there is no secondary current. Since $\mathbf{I^2R}$ loss also in the primary is very small the no load current is very small (usually 2 to 5 % of the full load current). The core loss is dependent on the flux which in turn depends on the applied voltage. Since full rated voltage is applied to the primary in this test full rated flux will be set up and the Full Core losses will be present and this will be constant at all loads. Since the $\mathbf{I^2R}$ loss in the primary is very small compared to the core losses they can be ignored and the full power consumed in the primary as read by wattmeter **W** can regarded as the core losses. With this understanding the readings of the various meters in the OC test are as follows.

- Ammeter reading : No load current $\mathbf{I_0}$
- Volt meter reading : Applied rated Primary voltage $\mathbf{V_1}$
- Wattmeter reading : Input power totally consumed as Core losses $\mathbf{P_{CL}}$

From these measurements the parameters $\mathbf{R_C}$ and $\mathbf{X_M}$ shown in the equivalent circuit can be computed as shown below.

No load power factor : $\mathbf{Cos\ \Theta = P_{CL} / V_1.I_0}$ **(Since Input power** $\mathbf{= P_{CL} = V_1.I_0. Cos\ \Theta}$ **)**

We already know from the equivalent circuit description that $\mathbf{I_{CL}}$ is the core loss current corresponding to $\mathbf{R_C}$ and $\mathbf{I_M}$ is the magnetizing current corresponding to $\mathbf{X_M}$ and in terms of the present measurements are given by :

$$I_{CL} = I_0. Cos\ \Theta \quad \text{and } I_M = I_0. Sin\ \Theta$$

Therefore $\mathbf{R_C}$ and $\mathbf{X_M}$ are given by :

$$R_C = V_1 / I_{CL} \quad \text{and } X_M = V_1 / I_M$$

Short Circuit test:

The test setup to conduct the SC test is shown in the figure below.

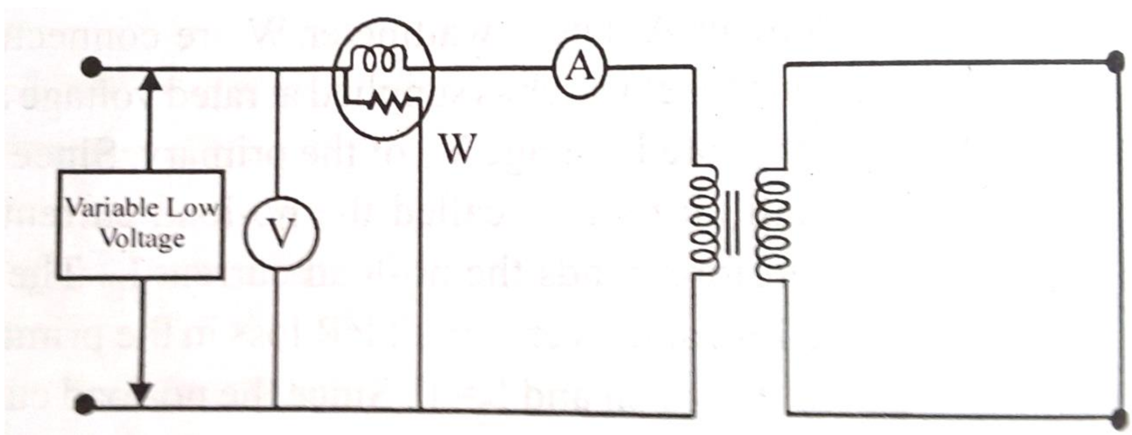


Fig: Test Setup to conduct the Short Circuit test

In this test high voltage side is designated as Primary (where an input voltage is applied) and the low voltage side is designated as Secondary which is usually short circuited by a thick conductor. (or sometimes through an Ammeter to read additionally the secondary load current) Voltmeter $\mathbf{V_1}$, Ammeter **A** and wattmeter **W** are connected in the primary as shown.
A very low voltage through a *Variac* (Variable auto transformer) is applied to the primary gradually from Zero Volts to about 5 to 10 % of the rated primary value till the primary current is just equal to the rated primary current. Since the secondary is short circuited we will get rated primary current with a low value of voltage itself. Since the applied voltage is very low the flux produced is also very low and hence the core losses also will be low and can be neglected. Now since the rated currents are flowing in both the Primary and the secondary the input power will be mostly consumed as copper losses. Since the secondary is short circuited the secondary voltage is zero and the entire input voltage $\mathbf{V_1}$ drops in the total equivalent impedance $\mathbf{Z_{EQP}}$ of the transformer reflected to the primary.

$$\text{i.e.} \quad V_1 = I_{PSC} . Z_{EQP}$$

The readings in the short circuit test are as follows:

- Ammeter reading : Primary current I_{PSC} (with secondary short circuited)
- Volt meter reading : Applied Primary voltage V_1
- Wattmeter reading : Input power totally consumed as Copper losses P_{CUL}

With this notation the power factor $\cos\Theta_{SC}$ in this test is given by:

$$P_{CUL} = V_1 \cdot I_{PSC} \cdot \cos\Theta_{SC}$$

From the above readings and the governing equations we can calculate the Equivalent Resistance, Equivalent Impedance and Equivalent Reactance parameters of the transformer referred to the primary as below.

Equivalent Resistance : $R_{EQP} = P_{CUL} / I^2_{PSC}$

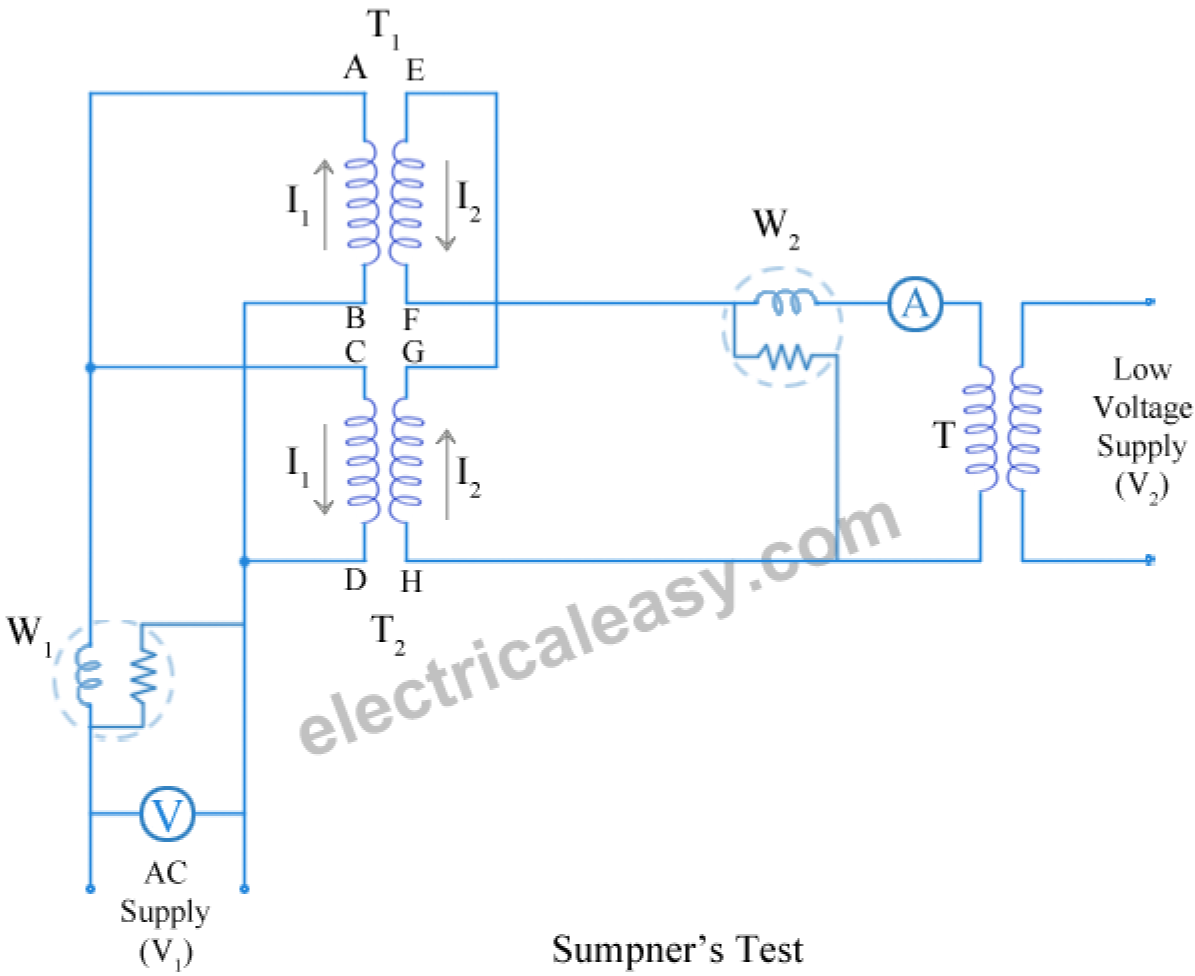
Equivalent Impedance : $Z_{EQP} = V_1 / I_{PSC}$

Equivalent Reactance : $X_{EQP} = \sqrt{Z^2_{EQP} - R^2_{EQP}}$

Sumpner's test or back to back test on transformer is another method for determining transformer efficiency, voltage regulation and heating under loaded conditions. Short circuit and open circuit tests on transformer can give us parameters of equivalent circuit of transformer, but they can not help us in finding the heating information. Unlike O.C. and S.C. tests, actual loading is simulated in Sumpner's test. Thus the Sumpner's test give more accurate results of regulation and efficiency than O.C. and S.C. tests.

Sumpner's Test

Sumpner's test or back to back test can be employed only when two identical transformers are available. Both transformers are connected to supply such that one transformer is loaded on another. Primaries of the two identical transformers are connected in parallel across a supply. Secondaries are connected in series such that emf's of them are opposite to each other. Another low voltage supply is connected in series with secondaries to get the readings, as shown in the circuit diagram shown below.



Sumpner's Test

In above diagram, T_1 and T_2 are identical transformers. Secondaries of them are connected in voltage opposition, i.e. E_{EF} and E_{GH}. Both the emf's cancel each other, as transformers are identical. In this case, as per superposition theorem, no current flows through secondary. And thus the no load test is simulated. The current drawn from V_1 is $2I_0$, where I_0 is equal to no load current of each transformer. Thus input power measured by wattmeter W_1 is equal to iron losses of both transformers.

i.e. iron loss per transformer Pi = $W_1/2$.

Now, a small voltage V_2 is injected into secondary with the help of a low voltage transformer. The voltage V_2 is adjusted so that, the rated current I_2 flows through the secondary. In this case, both primaries and secondaries carry rated current. Thus short circuit test is simulated and wattmeter W_2 shows total full load copper losses of both transformers.

i.e. copper loss per transformer $P_{Cu} = W_2/2$.

From above test results, the **full load efficiency of each transformer** can be given as:

$$\begin{array}{c}\text{\% full load efficiency} \\ \text{of each transformer}\end{array} = \frac{\text{output}}{\text{output} + \frac{W_1}{2} + \frac{W_2}{2}} \times 100$$

UNIT-III

Polyphase Transformers

- **Polyphase connections – Y/Y, Y/D, D/Y, D/D and open D, Third harmonics in phase voltages**
- **Three winding transformers Tertiary windings**
- **Determination of Zp, Zs and Zt transients in switching Off load and On load tap changing**
- **Scott connection.**

Introduction:

Almost all the major power generation and distribution systems in the world today are three-phase ac systems. Since three-phase systems play such an important role in modern life, it is necessary to understand how transformers are used in them.
Transformers for three-phase circuits can be constructed in one of two ways. One approach is simply to take three single-phase transformers and connect them in a three-phase bank. An alternative approach is to make a three-phase transformer consisting of three sets of windings wrapped on a common core.
These two possible types of transformer construction are shown in the figures below.
The construction of a single three-phase transformer is the preferred practice today, since it is lighter, smaller, cheaper, and slightly more efficient. The older construction approach was to use three separate transformers. That approach had the advantage that each unit in the bank could be replaced individually in the event of trouble, but that does not outweigh the ad vantages of a combined three phase unit for most applications. However, there are still a great many installations consisting of three single-phase units in service.
A discussion of three-phase circuits is included in Appendix A. Some readers may wish to refer to it before studying the following material.

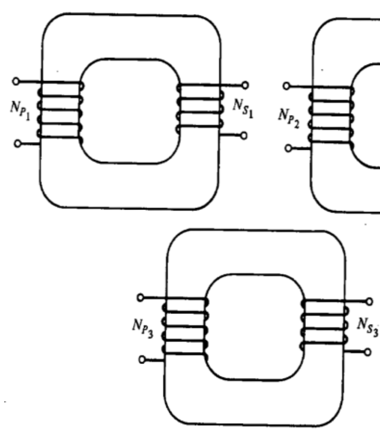


Fig: A three-phase transformer bank composed of independent transformers.

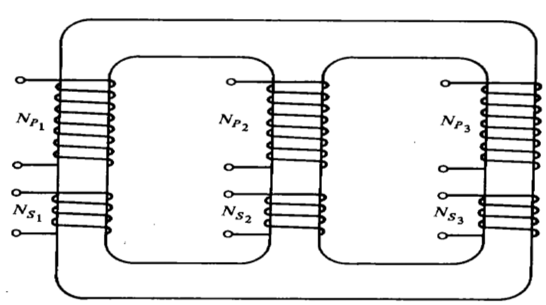


Fig: A three-phase transformer wound on a single three-legged core.

Three-Phase Transformer Connections:

A three-phase transformer consists of three transformers, either separate or combined on one core. The primaries and secondaries of any three-phase transformer can be independently connected in either a wye (Y) or a delta (Δ). This gives a total of four possible connections for a three-phase transformer bank:

I. Wye-wye (Y-Y)
2. Wye-delta (Y -Δ)
3. Delta-wye (Δ-Y)
4. Delta-delta (Δ-Δ)

The key to analyzing any three-phase transformer bank is to look at a single transformer in the bank. *Any single transformer in the bank behaves exactly like the single-phase transformers already studied.* the impedance, voltage regulation, efficiency, and similar calculations for three-phase transformers are done on *a per-phase basis,* using exactly the same techniques already developed for single-phase transformers.
The advantages and disadvantages of each type of three-phase transformer connection are explained below along with the relevant connection diagrams.

WYE-WYE CONNECTION:

The Y-Y connection of three-phase transformers is shown in the figure below.

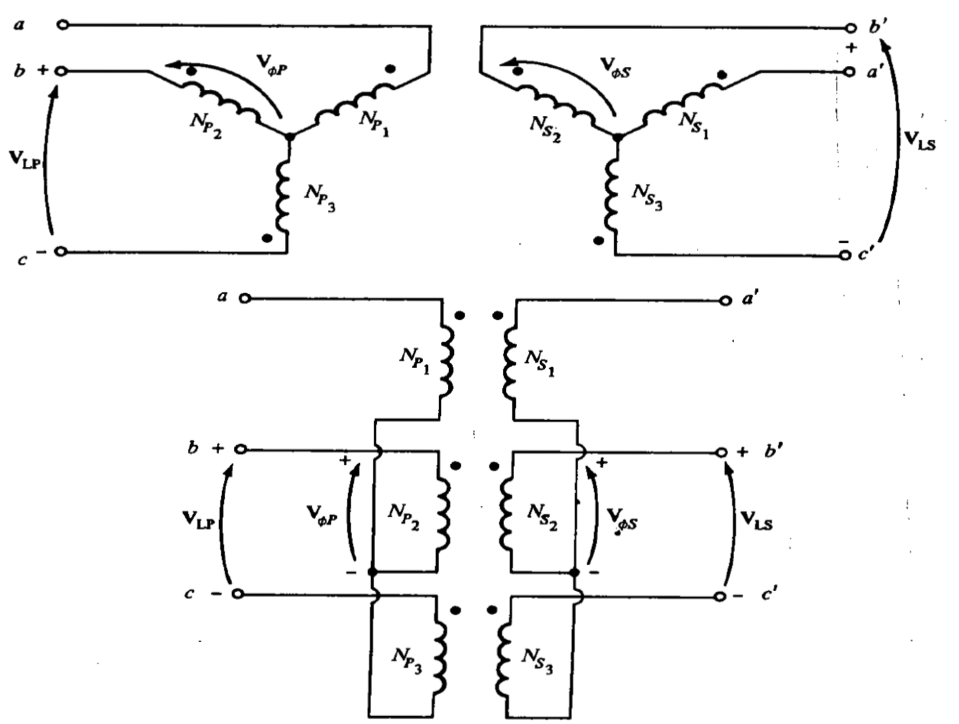


Fig: Three-phase transformer connections and wiring diagrams: Y-Y **connection**

In a Y-Y connection, the primary voltage on each phase of the transformer is given by $V_{\phi P} = V_{LP} / \sqrt{3}$. The primary-phase voltage is related to the secondary-phase voltage by the turns ratio of the transformer. The phase voltage on the secondary is then related to the line voltage on the secondary by $V_{LS} = \sqrt{3}V_{\phi S}$. Therefore, overall the voltage ratio on the transformer is

$$\frac{V_{LP}}{V_{LS}} = \frac{\sqrt{3}V_{\phi P}}{\sqrt{3}V_{\phi S}} = a$$

The Y-Y connection has two basic problems:
1. If loads on the transformer circuit are unbalanced, then the voltages on the phases of the transformer can become severely unbalanced.
2. Third-harmonic voltages can be large.
If a three-phase set of voltages is applied to a Y - Y transformer, the voltages in any phase will be 120^0 apart from the voltages in any other phase. However, *the third-harmonic components of each of the three phases will be in phase with each other,* since there are three cycles in the third harmonic for each cycle of the fundamental frequency. There are always some third-harmonic components in a transformer because of the nonlinearity of the core, and these components add up.
The result is a very large third-harmonic component of voltage on top of the 50 or 6O-Hz fundamental voltage. This third-harmonic voltage can be larger than the fundamental voltage itself.
Both the unbalance problem and the third-harmonic problem can be solved
using one of the two following techniques:

1.*Solidly ground the neutrals of the transformers,* especially the primary winding's neutral. This connection permits the additive third-harmonic components to cause a current flow in the neutral instead of building up large voltages. The neutral also provides a return path for any current imbalances in the load.
2. *Add a third (tertiary) winding connected in* Δ to the transformer bank. If a third Δ connected winding is added to the transformer, then the third-harmonic components of voltage in the Δwill add up, causing a circulating current flow within the winding. This suppresses the third-harmonic components of voltage in the same manner as grounding the transformer neutrals.
The Δ connected tertiary windings need not even be brought out of the transformer case, but they often are used to supply lights and auxiliary power within the substation where it is located. The tertiary windings must be large enough to handle the circulating currents, so they are usually made about
one-third the power rating of the two main windings.
One or the other of these correction techniques *must* be used any time a Y-Y transformer is installed. In practice, very few Y-Y transformers are used, since the same jobs can be done by one of the other types of three-phase transformers.

WYE-DELTA CONNECTION:

The Y - Δ connection of three-phase transformers is shown in the figure below. In this connection, the primary line voltage is related to the primary phase voltage by $V_{LP} = \sqrt{3}V_{\phi P}$ while the secondary line voltage is equal to the secondary phase voltage $V_{LS} = V_{\phi S}$. The voltage ratio of each phase is:

$$V_{\phi P} / V_{\phi S} = a$$

so the overall relationship between the line voltage on the primary side of the bank and the line voltage on the secondary side of the bank is:

$$V_{LP} / V_{LS} = \sqrt{3} V_{\phi P} / V_{\phi S} = \sqrt{3}\, a$$

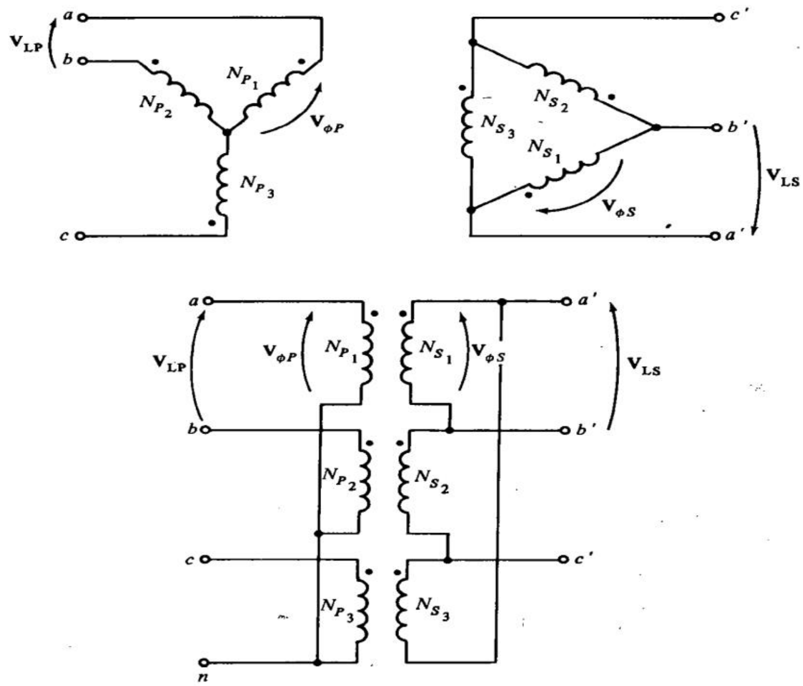


Fig: Three-phase transformer connections and wiring diagrams: Y - Δ **connection**

The Y - Δ connection has no problem with third-harmonic components in its voltages, since they are consumed in a circulating current on the Δ side. This connection is also more stable with respect to unbalanced loads, since the Δ partially redistributes any imbalance that occurs.

This arrangement does have one problem, though. Because of the connection, the secondary voltage is shifted 30^0 relative to the primary voltage of the transformer. The fact that a phase shift has occurred can cause problems in paralleling the secondaries of two transformer banks together. The phase angles of transformer secondaries must be equal if they are to be paralleled, which means that attention must be paid to the direction of the 30^0 phase shift occurring in each transformer bank to be paralleled together.

DELTA-WYE CONNECTION:

A Δ - Y connection of three-phase transformers is shown in the figure below. In a Δ - Y connection, the primary line voltage is equal to the primary-phase voltage $V_{LP} = V_{\phi P}$, while the secondary voltages are related by $V_{LS} = \sqrt{3} V_{\phi S}$ **.** Therefore, the line-to-line voltage ratio of this transformer connection is given by

$$\frac{V_{LP}}{V_{LS}} = \frac{V_{\phi P}}{\sqrt{3}V_{\phi S}}$$

$$\boxed{\frac{V_{LP}}{V_{LS}} = \frac{a}{\sqrt{3}} \qquad \Delta\text{-Y}}$$

is

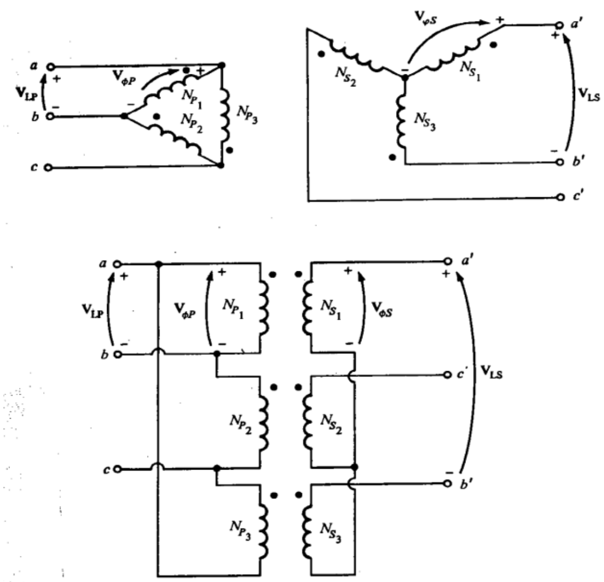


Fig: Three-phase transformer connections and wiring diagrams: Δ - Y **connection**

This connection has the same advantages and the same phase shift as the Y - Δ transformer. The connection shown in the figure above makes the secondary voltage lag the primary voltage by 30°, as before.

DELTA-DELTA CONNECTION:

The Δ- Δ connection is shown in the figure below.

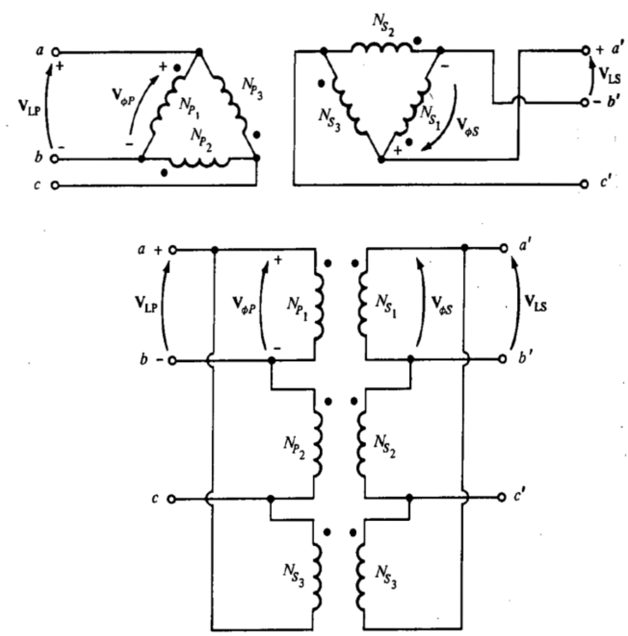


Fig: Three-phase transformer connections and wiring diagrams : Δ - Δ connection

In a Δ- Δ connection, $V_{LP} = V_{\phi P}$ and $V_{LS} = V_{\phi S}$, so the relationship between primary and secondary line voltages is given by:

$$\frac{V_{LP}}{V_{LS}} = \frac{V_{\phi P}}{V_{\phi S}} = a \qquad \Delta\text{-}\Delta$$

This transformer has no phase shift associated with it and no problems with unbalanced loads or harmonics.

The Scott-T Connection:

The Scott-T connection is a way to derive two phases 90° apart from a three-phase power supply. In the early history of ac power transmission, two-phase and three phase power systems were quite common. In those days, it was routinely necessary to interconnect two- and three-phase power systems, and the Scott-T transformer connection was developed for that purpose.

Today, two-phase power is primarily limited to certain control applications, but the Scott T is still used to produce the power needed to operate them.

The Scott T consists of two single-phase transformers with identical ratings. One has a tap on its primary winding at 86.6 percent of full-load voltage. They are connected as shown in the figure (a) below.

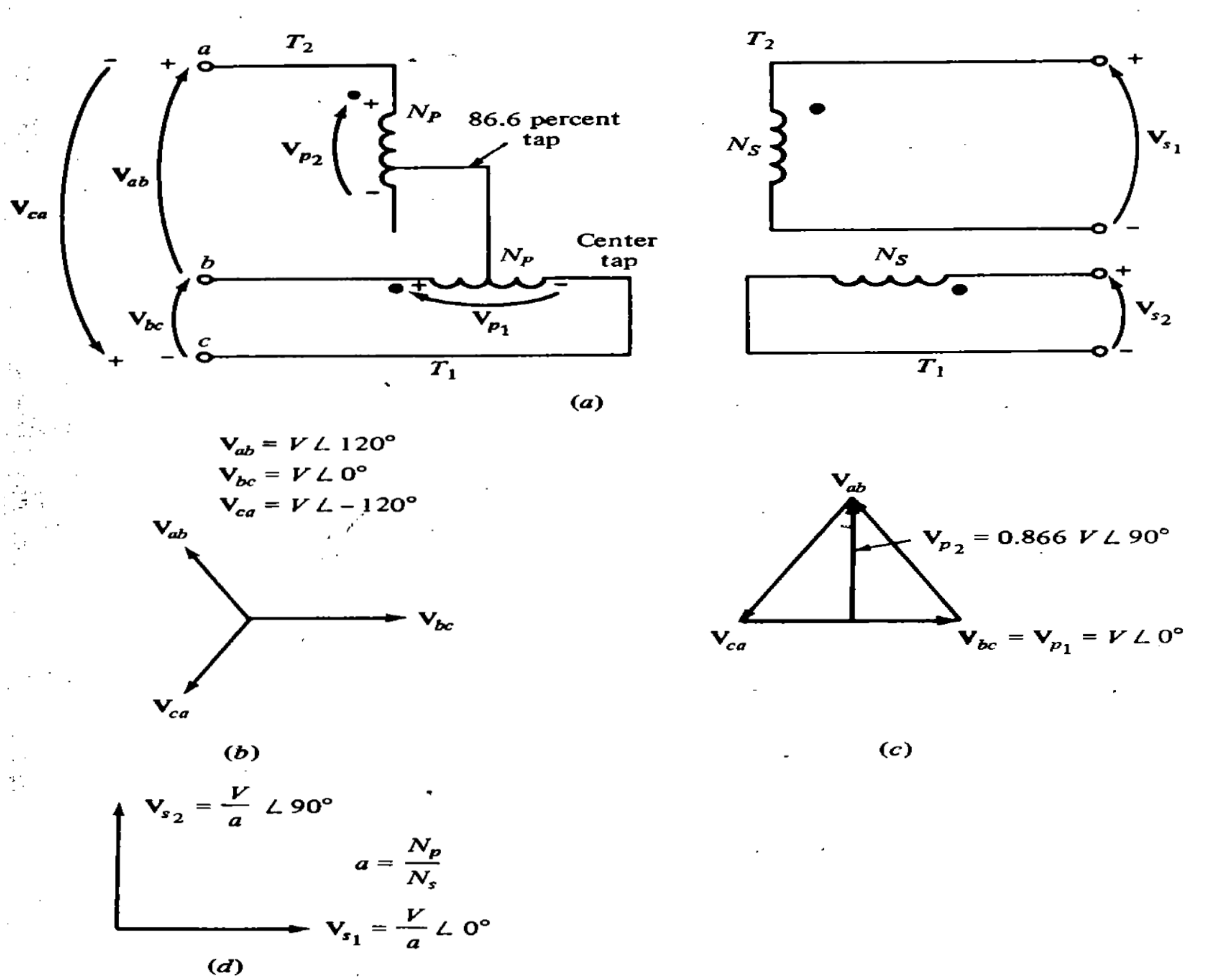


Fig: The Scott-T transformer connection. (a) Wiring diagram (b) the three-phase input voltages (c) the voltages on the transformer primary windings (d) the two-phase secondary voltages.

The 86.6 percent tap of transformer $\mathbf{T_2}$ is connected to the center tap of transformer $\mathbf{T_1}$. The voltages applied to the primary winding are shown in the figure (b), and the resulting voltages applied to the primaries of the two transformers are shown in the figure (c). Since these voltages are 90° apart, they result in a two-phase output.

It is also possible to convert two- phase power into three-phase power with this connection, but since there are very few two-phase generators in use, this is rarely done.

UNIT-IV

Polyphase Induction Motors

- **Construction details of cage and wound rotor machines**
- **Production of a rotating magnetic field**
- **Principle of operation**
- **Rotor emf and Rotor frequency**
- **Rotor reactance, rotor current and Power factor at standstill and during operation.**
- **Rotor power input, Rotor copper loss and mechanical power developed and their interrelation**
- **Torque equation – expressions for maximum torque and starting torque**
- **Torque slip characteristic**
- **Double cage and deep bar rotors**
- **Equivalent circuit – phasor diagram**
- **Crawling and Cogging**

Basic Induction Motor Concepts:

The Development of Induced Torque in an Induction Motor:

When current flows in the stator, it will produce a magnetic field in stator as such that **Bs** (stator magnetic field) will rotate at a speed:

$$n_S = 120.f_S/P$$

Where *fs* is the system frequency in hertz and **P** is the number of poles in the machine. This rotating magnetic field **Bs** passes over the rotor bars and induces a voltage in them. The voltage induced in the rotor is given by:

$$e_{ind} = (v \times B)\, l$$

Where **v** is the velocity of the Rotor bars relative to the Stator magnetic field

B = magnetic flux density vector

And **l =** length of the rotor bar in the magnetic field.

Hence there will be rotor current flow which would be lagging due to the fact that the rotor is Inductive. And this rotor current will produce a magnetic field at the rotor, **Br**. Hence the Interaction between these two magnetic fields would give rise to an induced torque:

$$T_{ind} = k.B_R \times B_S$$

The torque induced would accelerate the rotor and hence the rotor will rotate .

However, there is a finite upper limit to the motor's speed due to the following interactive phenomenon:

If the induction motor's speed increases and reaches synchronous speed then the rotor bars would be stationary relative to the magnetic field

↓

No induced voltage

↓

No rotor current

↓

No rotor magnetic field

↓

Induced torque = 0

↓

Rotor will slow down due to friction

Conclusion: An induction motor can thus speed up to such a near synchronous speed where the induced torque is just able to overcome the load torque but it can never reach synchronous speed.

The Concept of Rotor Slip:

The induced voltage in the rotor bar is dependent upon the *relative speed between the stator Magnetic field and the rotor*. This is termed as slip speed and is given by:

$$n_{slip} = n_{sync} - n_m$$

Where n_{slip} = slip speed of the machine

n_{sync} = speed of the magnetic field (also motor's synchronous speed)and

n_m = mechanical shaft speed of the motor.

Apart from this we can describe this relative motion by using the concept of ***slip*** which is the relative speed expressed on a per-unit or percentage basis. ***Slip S*** is defined as

$$s = \frac{n_{slip}}{n_{sync}} (\times\ 100\%)$$

$$s = \frac{n_{sync} - n_m}{n_{sync}} (\times\ 100\%)$$

On percentage basis and is defined as

$$S = (N_{sync} — N_m) / N_{sync}$$

On per unit basis.

Slip **S** is also expressed in terms of angular velocity ***ω*** (Rad/Sec) as given below:

$$s = \frac{\omega_{sync} - \omega_m}{\omega_{sync}} (\times\ 100\%)$$

It can be noted that if the motor runs at synchronous speed the slip ***S*** =0 and if the rotor is standstill then the slip ***S*** =1.
It is possible to express the mechanical speed of the Rotor in terms of Slip ***S*** and synchronous speed n_{sync} as given below:

$$n_m = (1 - s)n_{sync}$$

$$\omega_m = (1 - s)\omega_{sync}$$

The Electrical Frequency on the Rotor:
An induction motor is similar to a rotating transformer where the primary is similar to the stator and the secondary a rotor**.** But unlike a transformer, the secondary frequency may not be the same as in the primary. If the rotor is locked (cannot move), the rotor would have the same frequency as the stator. Another way to look at it is to see that when the rotor is locked, rotor speed drops to zero, hence slip is 1. But as the rotor starts to rotate, the rotor frequency would reduce, and when the rotor runs at synchronous speed, the frequency on the rotor will be zero. For any speed in between, the rotor frequency is directly proportional to the difference between the speed of the magnetic field n_{sync} and speed of the rotor n_m.Since slip of the rotor ***S*** is defines as :

$$S = (n_{sync} - n_m)/n_{sync}$$

Hence the rotor frequency can be expressed as :

$$f_r = s.f_s$$

Substituting the value of **S** above in the expression for f_r we get

$$f_r = (n_{sync} - n_m).f_s / n_{sync}.$$

And then substituting the value of f_s from the earlier relation $n_S = 120.f_S/P$

We get

$$f_r = (P/120).\ (n_{sync} - n_m.)$$

Development of Equivalent Circuit of an Induction Motor:

An induction motor relies for its operation on the induction of voltages and currents in its rotor circuit from the stator circuit (transformer action). This induction is essentially a transformer operation, and hence the equivalent circuit of an induction motor is developed starting with that of a transformer.

The Transformer Model of an Induction Motor

A transformer per-phase equivalent circuit, representing the operation of an induction motor is shown below:

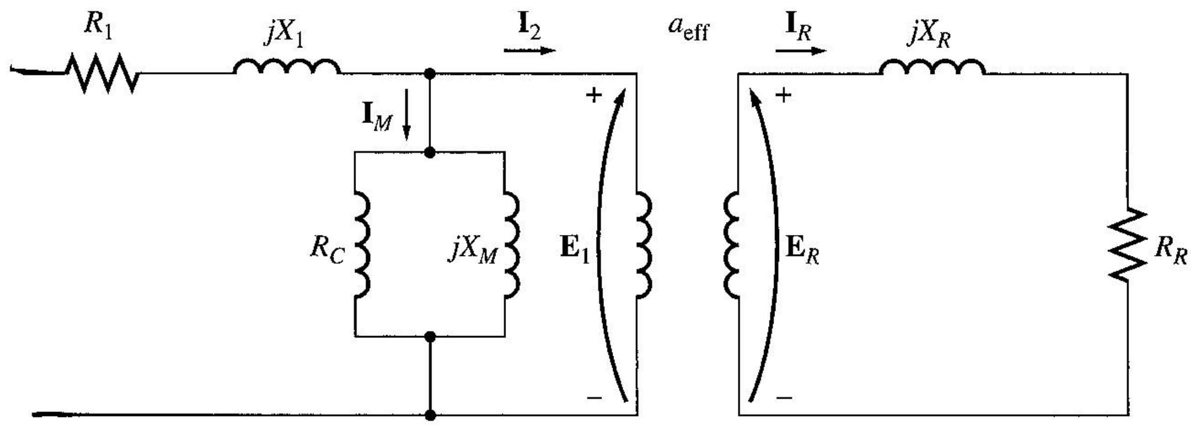


Fig:The transformer model or an induction motor, with rotor and stator connected by an ideal transformer of turns ratio *a*eff.

As in any transformer, there is certain resistance and self-inductance in the primary (stator) windings, which are represented in the equivalent circuit of the machine. They are R1-stator resistance and X1-stator leakage reactance

Also, like any transformer with an iron core, the flux in the machine is related to the integral of the applied voltage E1. The curve of mmf vs. flux (magnetization curve) for this machine is compared to that of a transformer, as shown below:

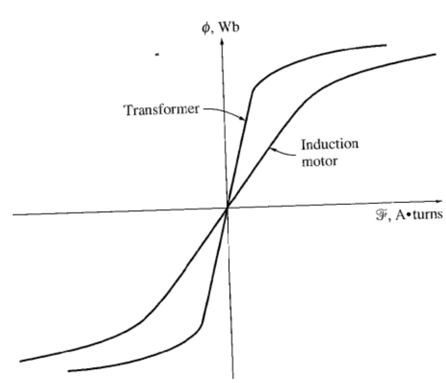


Fig: The magnetisation characteristics of a Transformer vs. Induction motor.

The slope of the induction motor's mmf-flux curve is much shallower than the curve of a good transformer. This is because there must be an air gap in an induction motor, which greatly increases the reluctance of the flux path and thus reduces the coupling between primary and secondary windings. The higher reluctance caused by the air gap means that a higher magnetizing current is required to obtain a given flux level. Therefore, the magnetizing reactance X_m in the equivalent circuit will have a much smaller value than that in a transformer.

The primary internal stator voltage is E_1 is coupled to the secondary E_R by an ideal transformer with an effective turns ratio a_{eff}. The turns ratio for a wound rotor is basically the ratio of the conductors per phase on the stator to the conductors per phase on the rotor. It is rather difficult to see a_{eff} clearly in a cage rotor because there are no distinct windings on the cage rotor.

E_R in the rotor produces a current flow in the shorted rotor (or secondary) circuit of the machine. The primary impedances and the magnetization current of the induction motor are very similar to the corresponding components in a transformer equivalent circuit.

The Rotor Circuit Model:

When the voltage is applied to the stator windings, a voltage is induced in the rotor windings. In general, the greater the relative motion between the rotor and the stator magnetic fields, the greater is the resulting rotor voltage and rotor frequency**.** The largest relative motion occurs when the rotor is stationary, called the *locked-rotor* or *blocked-rotor* condition, so the largest voltage and rotor frequency are induced in the rotor at that condition. The smallest voltage and frequency occur when the rotor moves at the same speed as the stator magnetic field, resulting in no relative motion.

The magnitude and frequency of the voltage induced in the rotor at any speed between these extremes is directly proportional to the slip of the rotor. Therefore, if the magnitude of the induced rotor voltage at locked-rotor conditions is taken as E_{R0}, then the magnitude of the induced voltage at any slip will be given by:

$$E_R = s.E_{R0}$$

This voltage is induced in a rotor containing both resistance and reactance. The rotor resistance R_R is a constant, independent of slip, while the rotor reactance is affected in a more complicated way by slip.The reactance of an induction motor rotor depends on the inductance of the rotor and the frequency of voltage and current in the rotor. With a rotor inductance of L_R, the rotor reactance X_R is given by :

$$X_R = \omega_r L_R = 2\pi f_r L_R$$

$$\text{Since } f_r = sf_s$$

$$X_R = s.2\pi f_s L_R = sX_{R0}$$

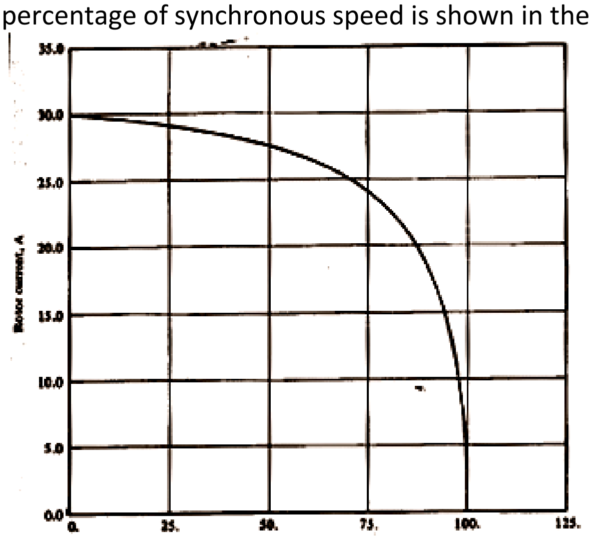
Where X_{R0} is the blocked rotor reactance. The resulting rotor equivalent circuit is as shown below:

The rotor current is given by :

$$I_R = E_R / (R_R + jX_R)$$
$$I_R = s.E_{RO} / (R_R + s.jX_{RO})$$
$$I_R = E_{R0} / (R_R/s + jX_{R0})$$

In the given expression for the rotor current it can be seen that all the effects on **rotor** of varying rotor speed are reflected in the varying impedance **Z_{Req} = (R_R/s + jX_{R0})** supplied from a constant voltage source **E_{R0}**.
In this modified equivalent circuit shown below , the rotor voltage is a constant **E_{R0}** and the rotor

impedance contains all the effects of varying rotor slip. Based upon the equation above, at low slips, it can be seen that the rotor resistance is much larger in magnitude as compared to **X_{R0}**. At high slips, **X_{R0}** will be larger as compared to the rotor resistance. Based on the above equation for the rotor current a plot of **I_R** as a function of percentage of synchronous speed is shown in the figure below.



% of synchronous speed

Fig: Rotor current as a function of Rotor speed.

To produce the final per-phase equivalent circuit for an induction motor, it is necessary to refer the rotor part of the model over to the stator side. In an ordinary transformer, the voltages, currents and impedances on the secondary side can be referred to the primary by means of the turns ratio of the transformer:

$$V_P = V'_S = aV_S$$

$$I_P = I'_S = \frac{I_S}{a}$$

and $$Z'_S = a^2 Z_S$$

Exactly the same sort of transformation can be done for the induction motor's rotor circuit. If the Effective turns ratio of an induction motor is $\mathbf{a_{eff}}$, then the transformed rotor voltage becomes:

$$\mathbf{E_S = E'_R = a_{eff} . E_{RO}}$$

The rotor current becomes: $\mathbf{I_2 = I_R / a_{eff}}$ and the Rotor impedance becomes :

$$\mathbf{Z_2 = a_{eff}^2 .(R_R/s + jX_{R0})}$$

And if we give the following definitions **:**

$$\mathbf{R_2 = a_{eff}^2 . R_R}$$

$$\mathbf{X_2 = a_{eff}^2 . X_{RO}}$$

Then the final per- phase equivalent circuit of an Indcution motor would become as shown in the figure below.

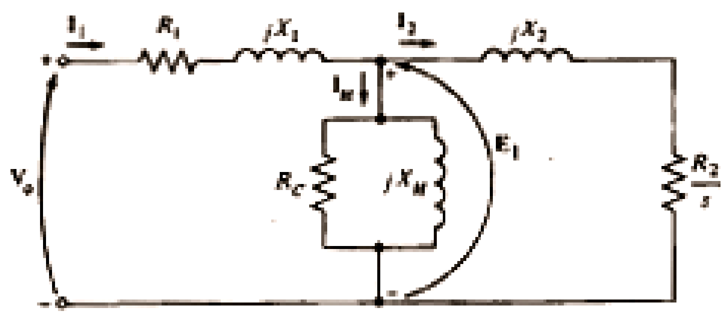


Fig: Final per-phase equivalent circuit of an induction motor.

For ease of calculating the motor current and the developed torque the magnetising reactance X_m is moved to the input side assuming that the drop across the stator resistance is small and the resulting final simplified equivalent circuit is shown in the figure below.

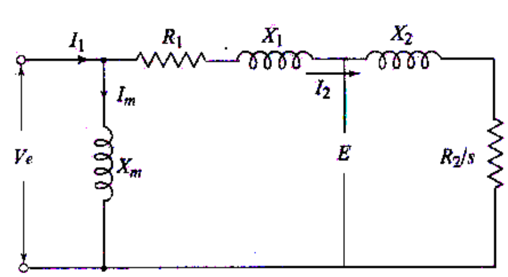


Fig: Fianl Simplified Per-phase equivalent circuit of an Induction Motor

Power and Torque in Induction Motor:

Having developed the simplified equivalent circuit of an Induction Motor we will now look at the power flow & losses in an Induction motor and then derive the expressions for the Motor current ,developed torque, Starting torque etc and the relation between Torque and power

Losses and Power-Flow diagram:

An induction motor can be basically described as a rotating transformer. Its input is a 3 phase system of voltages and currents. For an ordinary transformer, the output is electric power from the secondary windings. The secondary windings in an induction motor (the rotor) are shorted out, so no electrical output exists from normal induction motors. Instead, the output is mechanical. The relationship between the input electric power and the output mechanical power of this motor is shown below in the power flow diagram:

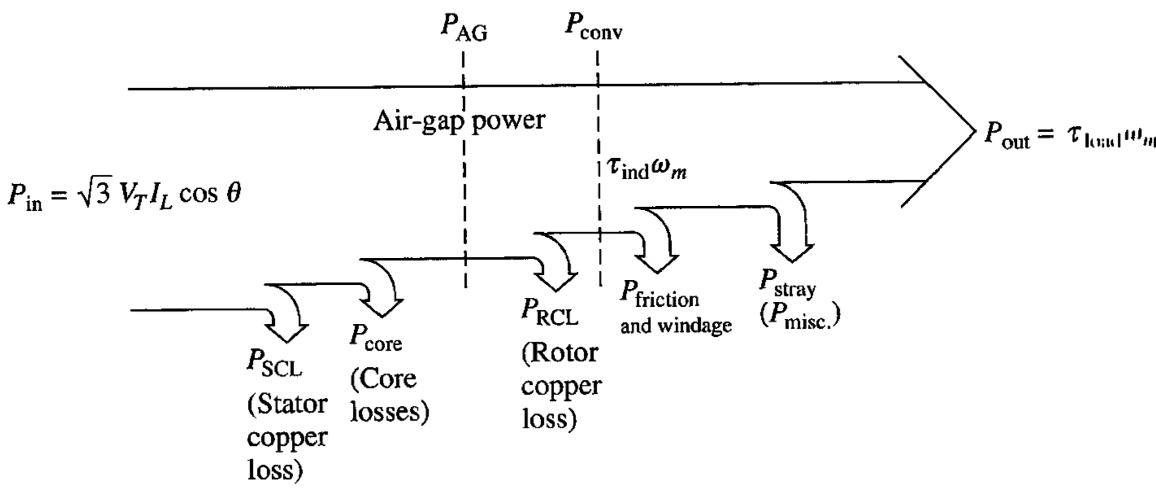


Fig: Power flow diagram of an Induction motor.

The input power to an induction motor P_{in} is in the form of 3-phase electric voltages and currents. The first losses encountered in the machine are I^2R losses in the stator windings (the stator copper loss P_{SCL}). Then, some amount of power is lost as hysteresis and eddy currents in the stator (P_{core}). The power remaining at this point is transferred to the rotor of the machine across the air gap between the stator and rotor. This power is called the air gap power P_{AG} of the machine. After the power is transferred to the rotor, some of it is lost as I^2R losses (the rotor copper loss P_{RCL}), and the rest is converted from electrical to mechanical form (P_{conv}). Finally, friction and windage losses $P_{F\&W}$ and stray losses P_{misc} are subtracted. The remaining power is the output of the motor P_{out}.

The core losses do not always appear in the power-flow diagram at the point shown in the figure above.Because of the nature of the core losses, where they are accounted for in the machine is somewhat arbitrary. The core losses of an induction motor come partially from the stator circuit and partially from the rotor circuit. Since an induction motor normally operates at a speed near synchronous speed, the relative motion of the magnetic fields over the rotor surface is quite slow, and the rotor core losses are very tiny compared to the stator core losses. Since the largest fraction of the core losses comes from the stator circuit, all the core losses are lumped together at that point on the diagram.

These losses are represented in the induction motor equivalent circuit by the resistor $\mathbf{R_C}$ (or the conductance $\mathbf{G_C}$). If core losses are just given by a number (X watts) instead of as a circuit element, they are often lumped together with the mechanical losses and subtracted at the point on the diagram where the mechanical losses are located.
The *higher* the speed of an induction motor, the *higher* the friction, windage, and stray losses. On theother hand, the *higher* the speed of the motor (up to *nsync*), the *lower* its core losses. Therefore, these three categories of losses are sometimes lumped together and called *rotational losses*. The total rotational losses of a motor are often considered to be constant with changing speed, since the component losses change in opposite directions with a change in speed.

Power and Torque in an Induction Motor:

By examining the per-phase equivalent circuit, the power and torque equations governing the operation of the motor can be derived.
The input current to one phase of the motor is given by :

$$I_1 = V_\phi/Z_{eq}$$

Thus by finding out $\mathbf{Z_{eq}}$ and $\mathbf{I_1}$,the stator copper losses, the core losses, and the rotor copper losses can be found out.
The stator copper losses in the 3 phases are: $P_{SCL} = 3\, I_1^2\, R_1$
The core losses are: $P_{Core} = 3\, E_1^2\, G_C$
And the air gap power is: $P_{AG} = P_{in} - P_{SCL} - P_{core}$
Also, the only element in the equivalent circuit where the air-gap power can be consumed is in the Resistor **R2/s**. Thus, the air-gap power is given by:

$$P_{AG} = 3I_2^2.R_2/s$$

The actual resistive losses in the rotor circuit are given by:

$P_{RCL} = 3\, I_R^2\, R_R$ (I_R& R_R are the rotor current and resistance before referring to the stator side)
Since power is unchanged when referred across an ideal transformer, the rotor copper losses can also be expressed as:

$$P_{RCL} = 3\, I_2^2\, R_2$$

And the rotor copper losses are noticed to be equal to slip times the air gap power i.e. $P_{RCL} = s\,.P_{AG}$

After stator copper losses & core losses, rotor copper losses are subtracted from the input power to the motor, to get the remaining power which is converted from electrical to mechanical form. The power thus converted, which is called developed (converted) mechanical power is given as:

$$P_{conv} = P_{AG} - P_{RCL}$$
$$= 3I_2^2.R_2/s - 3I_2^2R_2$$
$$= 3I_2^2\,[R_2(1/s) - 1]$$
$$P_{conv} = 3I_2^2\,[R_2(1-s)/s]$$

Hence, the lower the slip of the motor, the lower the rotor losses. Also, if the rotor is not running, the slip is **s=1** and the air gap power is entirely consumed in the rotor. This is logical, since if the rotor is not running the output power P_{out} ($= \tau_{load}\,.\,\omega_m$) must be zero. Since $P_{conv} = P_{AG} - P_{RCL}$, this also gives another relationship between the air-gap power and the power converted from electrical to mechanical form:

$$P_{conv} = P_{AG} - P_{RCL}$$
$$= P_{AG} - sP_{AG}$$

$$\mathbf{P_{conv} = (1\text{-}s)\, P_{AG}}$$

Finally, if the friction,windage and the stray losses are known, the output power:

$$\mathbf{P_{out} = P_{conv} - P_{F\&W} - P_{stray}}$$

The induced torque in a machine was defined as the torque generated by the internal electric to mechanical power conversion. This torque differs from the torque actually available at the terminals of the motor by an amount equal to the friction and windage torques in the machine. Hence, the developed torque is given by :

$$\mathbf{T_{ind} = P_{conv} / \omega_m}$$

And the other ways to express the torque is :

$$\mathbf{T_{ind} = (1-s)P_{AG} / (1-s)\omega_s}$$

$$\mathbf{T_{ind} = P_{AG} / \omega_s}$$

From the above study and the developed simplified equivalent circuit the rotor current is given by

$$I_2 = \frac{V_1}{(R_1 + R_2/s) + j(X_1 + X_2)}$$

$$I_2 = \frac{V_1}{\sqrt{(R_1 + R_2/s)^2 + (X_1 + X_2)^2}}$$

Now, the gross converted mechanical power ***P*** is given by:

$$3I_2^2 R_2 (1-s)/s = \frac{3V_1^2 R_2 (1-s)/s}{(R_1 + R_2/s)^2 + (X_1 + X_2)^2}$$

The developed torque is then given by :

$$T_d = \frac{P_{gross}}{\omega_r} = \frac{P_{gross}}{\omega_s(1-s)} = \frac{3V_1^2 R_2/s}{\omega_s[(R_1 + R_2/s)^2 + (X_1 + X_2)^2]}$$

$$\text{or} \qquad T_d = \frac{3}{\omega_s} I_2^2 \frac{R_2}{s} \text{ N-m}$$

As can be seen, in this equation the slip is the variable. Hence the maximum torque is obtained by taking derivative of the torque with respect to the slip and then setting the derivative to zero. Then we get the slip at maximum torque as

$$S_{max\,T} = \pm \frac{R_2}{\sqrt{R_1^2 + (X_1 + X_2)^2}}$$

And substituting this value of S_{maxT} in the above expression for developed torque we get the maximum developed torque T_{max} as:

$$T_{max} = \frac{3V_{1ph}^2}{2\omega_s\left[R_1 \pm \sqrt{R_1^2 + (X_1 + X_2)^2}\right]}$$

Here while working out problems we have to take the per phase voltage applied to the Induction motor by carefully looking at the input voltage and rotor winding connections.

Induction Motor Torque-Speed Characteristics:

The torque-speed relationship will be examined from the physical viewpoint of the motor's magnetic field behaviour .Then, a general equation for torque as a function of slip will be derived from the Induction motor equivalent circuit.

Induced Torque from a Physical Standpoint:

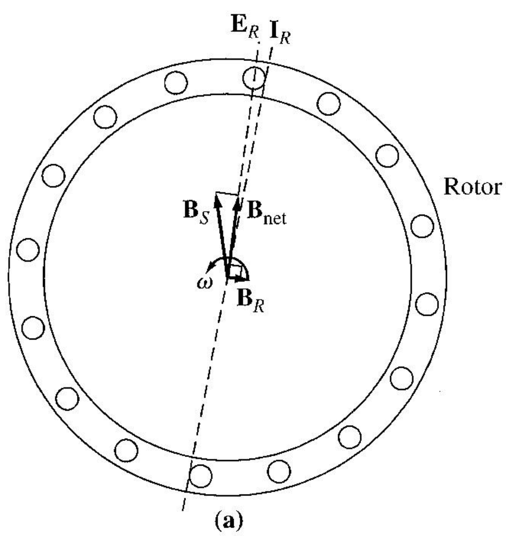


Fig: The magnetic fields in an induction motor under (a) light loads. (b) heavy loads

No-load Condition :

Assume that the induction motor is already rotating at no load conditions:

- Its rotating speed is near to synchronous speed. The net magnetic field B_{net} is produced by the magnetization current I_M .
- The magnitude of I_M and B_{net} is directly proportional to voltage E_1 . If E_1 is constant, then B_{net} is constant.
- In an actual machine, E_1 varies as the load changes due to the stator impedances R_1 and X_1 which cause varying volt drops with varying loads. However, the volt drop at R_1 and X_1 is so small, that E_1 can be assumed to remain constant throughout.
- At no-load, the rotor slip is very small, so the relative motion between rotor and magnetic field is very small, and hence the rotor frequency is also very small.
- Since the relative motion is small, the voltage E_R induced in the bars of the rotor is also very small, and hence the resulting current flow I_R is also very small.

- Since the rotor frequency is small, the reactance of the rotor is nearly zero, and the max rotor current $\mathbf{I_R}$ is almost in phase with the rotor voltage $\mathbf{E_R}$.
- The rotor current produces a small magnetic field $\mathbf{B_R}$ at an angle slightly greater than 90 degrees behind $\mathbf{B_{net}}$.
- The stator current will be quite large even at no-load since it must supply most of B_{net}.

The induced torque which keeps the rotor running is given by:

$$T_{ind} = kB_R \times B_{net}$$

And its magnitude is:

$$T_{ind} = kB_r B_{net} \sin\delta$$

In terms of magnitude, the induced torque will be small due to small rotor magnetic field.

On-load Conditions:

As the motor's load increases, its slip increases, and the rotor speed falls. Since the rotor speed is slower,there is now more relative motion between rotor and stator magnetic fields.

- Greater relative motion means a stronger rotor voltage $\mathbf{E_R}$ which in turn produces a larger rotor current $\mathbf{I_R}$.
- With large rotor current, the rotor magnetic field B_R also increases. However, the angle between rotor current and $\mathbf{B_R}$ changes as well.
- Since the rotor slip is larger, the rotor frequency rises **(*$f_r = sf_e$*)** and the rotor reactance increases **(ωL_R).**
- Therefore, the rotor current now lags further behind the rotor voltage, and the rotor magnetic field shifts with increasing load current.
- The rotor current now has increased compared to no-load but the angle **δ** has also increased. The increase in $\mathbf{B_R}$ tends to increase the torque, while the increase in angle **δ** tends to decrease the torque ($\mathbf{T_{ind}}$ is proportional to sin **δ**, and **δ>90º**).
- Since the first effect is larger than the second one, the overall induced torque increases to supply the motor's increased load.
- But as the load on the shaft is increased further , the **Sin δ** term decreases more than the $\mathbf{B_R}$ term increases (the value is going towards the 0 cross over point for a sine wave). At that point, a further increase in load decreases $\mathbf{T_{ind}}$ and the motor stops. The torque at which this happens is known as **pullout torque**.

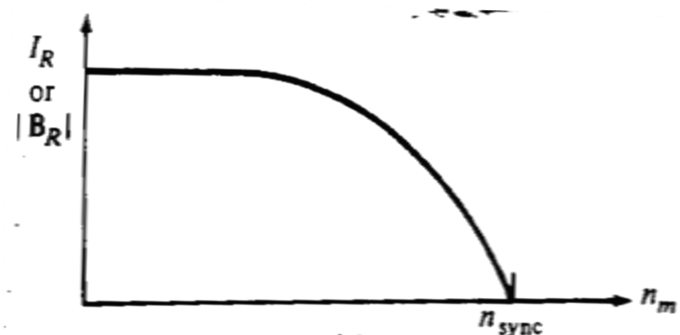
Developing the torque-speed characteristics of an induction motor:

As we have already seen the magnitude of the induced torque in the Induction motor is given by:

$$T_{ind} = kB_r B_{net} \sin\delta$$

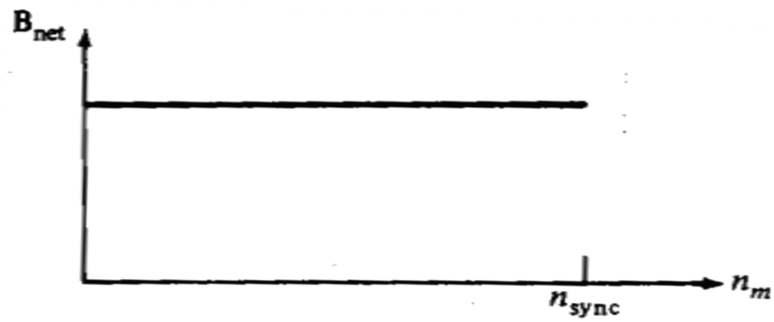
From the motor behaviour from no load to full load as explained above, the overall torque speed characteristics can be developed by considering each of the terms in the above expression for torque .

a) $\mathbf{B_r}$: Rotor magnetic field is directly proportional to the rotor current and will increase as the rotor current increases(Assuming that the rotor core is not saturated). The current flow will increase as slip increases (reduction in speed).The current flow as a function of motor speed is shown in fig(a) below.



(a)

b) B_{net}: The net magnetic field density **B_{net}** will almost remain constant since it is proportional to **E1** (refer to the induction motor equivalent circuit) and **E1** is assumed to be constant). The variation of **B_{net}** as a function of motor speed is shown in the fig(b) below.



(b)

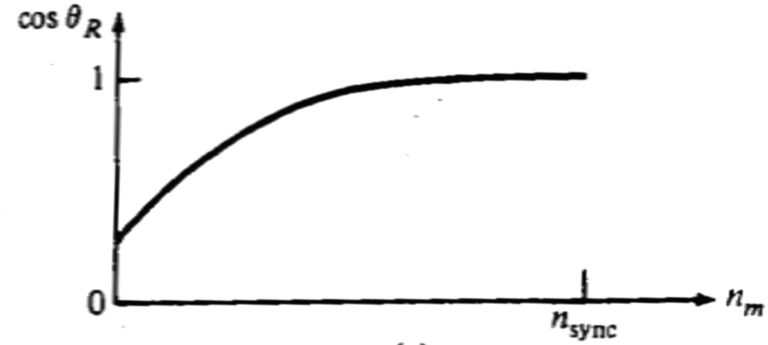
c)Sin δ :The angle **δ** between the Net and the Rotor magnetic fields can be expressed in a useful way. Looking at the figure above (magnetic fields on no-load and load) it can be seen that the angle **δ** is equal to the sum of the Rotor power factor angle **θ_r** and 90°(where **θ_r** is the angle between E_R and I_R. (Also note that E_R is in phase with B_{net} since it is in phase with B_{net}).

$$\text{i.e. } \delta = \theta_r + 90° \text{ and}$$
$$\sin\delta = \sin(\theta_r + 90°) = \cos\theta_r$$

Cos θ_R is also known as the motor power factor where:

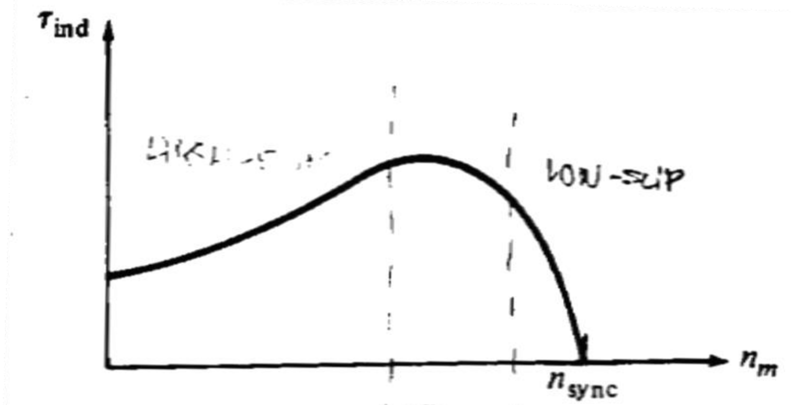
$$\theta_r = \tan^{-1}(X_r/R_r) = \tan^{-1}(sX_0/R_r)$$

A plot of Rotor power factor vs. Slip is shown in fig.(c) below.



(c)

Since the Induced torque is proportional to the product of the above three terms the total Torque speed characteristics of the Motor can be derived by graphical multiplication of the above three plots and is shown in fig(d) below.



(d)

The detailed Torque speed characteristics of an a Induction Motor Showing the Starting, Pull-out and Full-load torques are shown in the figure below.

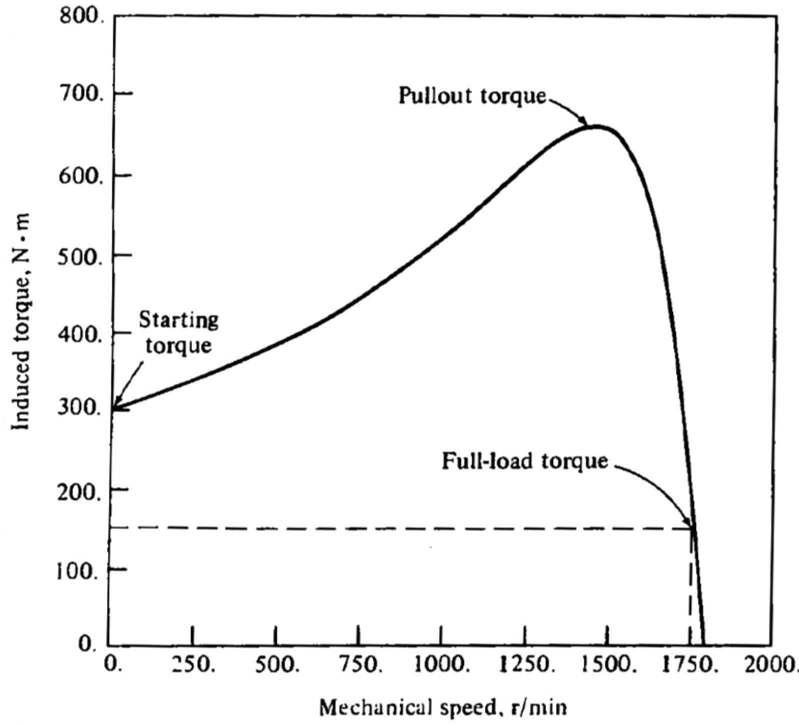


Fig: Torque speed characteristics of an a Induction Motor Showing the Starting, Pull-out and Full-load torques

Summary:

Important concepts and conclusions:

- Induction motor works on the principle of induction from stator rotating magnetic field to the rotor.
- The magnitude of the induced Torque in an Induction motor is given by :

$$T_{ind} = kB_r B_{net} \sin\delta$$

- ***The Torque speed characteristic can be divided into three important regions*:**

1. Low Slip Region: In this region :

- The motor slip increases approximately linearly with increased load.
- The mechanical speed decreases approximately linearly with increased load.
- Rotor reactance is negligible. So Rotor Power factor is almost unity.
- Rotor current increases linearly with slip.

The entire normal steady state operating range of an Induction motor is included in this linear low slip region. Thus in normal operation an induction motor has a linear speed drooping characteristic

2. Moderate slip region: In this region:

- Rotor frequency is higher than earlier and hence the Rotor reactance is of the same order of magnitude as the rotor resistance.

- Tor current no longer increases as rapidly as earlier and the Power factor starts dropping.
- The peak torque(Pull out or Break down Torque) occurs at a point where for an incremental increase in load the increase in the current is exactly balanced by the decrease in rotor power factor.

3. High slip region: In this region:

- The induced torque actually decreases with increase in load torque since the increase in Rotor current is dominated by the decrease in Rotor power factor.

- **Important characteristics of the Induction Motor Torque Speed Curve:**

  - Induced Torque is zero at synchronous speed.
  - The graph is nearly linear between no load and full load (at near synchronous speeds).In this region the Rotor resistance is much larger than the Rotor reactance ,and hence the Rotor Current, magnetic field and the induced torque increases linearly with increasing slip.
  - There is a Max. Possible torque that cannot be exceeded which is known as pull out torque or breakdown torque. This is normally about two to three times the full load torque.
  - The Starting torque is higher than the full load torque and is about 1.5 times. Hence this motor can start with any load that it handle at full power.
  - Torque for a given slip varies as the square of the applied voltage. This fact is useful in the motor speed control with variation of Stator Voltage.
  - If the rotor were driven faster than synchronous speed, then the direction of the Induced torque would reverse and the motor would work like a generator converting mechanical power to Electrical power.
  - If we reverse the direction of the stator magnetic field, the direction of the induced torque in the Rotor with respect to the direction of motor rotation would reverse, would stop the motor rapidly and will try to rotate the motor in the other direction. Reversing the direction of rotation of the magnetic field is just phase reversal and this method of Braking is known Plugging

Important formulae and equations:

- Synchronous speed of rotating magnetic field : $\mathbf{n_S = 120.f_S/P}$
- Volatge induced in the rotor : $\mathbf{e_{ind} = (v \times B)\, l}$
- Torque induced in the rotor : $\mathbf{T_{ind} = k.B_R \times B_S}$
- Magnitude of the Torque induced in the Rotor : $\mathbf{T_{ind} = kB_r B_{net} \sin\delta}$
- slip **s** on percentage basis:

$$s = \frac{n_{\text{slip}}}{n_{\text{sync}}} (\times\ 100\%)$$

$$\boxed{s = \frac{n_{\text{sync}} - n_m}{n_{\text{sync}}} (\times\ 100\%)}$$

- Slip **s** on per unit basis: $\mathbf{S = (N_{sync} - N_m) / N_{sync}}$

- The magnitude of the rotor induced voltage $\mathbf{E_R}$ in terms of the rotor induced voltage at rotor locked condition $\mathbf{E_{R0}}$: $\mathbf{E_R = s.E_{R0}}$
- The magnitude of the rotor Reactance $\mathbf{X_R}$ in terms of the rotor Reactance at rotor locked condition $\mathbf{X_{R0}}$: $\mathbf{X_R = s.X_{R0}}$ (since $f_r = s.f_S$ and $X_R = s.2\pi f_S L_R$)
- The rotor frequency can be expressed as :

$$\mathbf{f_r = (P/120).(n_{sync} - n_m.)}$$

- Important relationships between Air gap power $\mathbf{P_{AG}}$, converted power $\mathbf{P_{conv}}$, Rotor induced Torque $\mathbf{T_{ind}}$, Rotor copper losses $\mathbf{P_{rcl}}$ and the slip **s :**

$$\mathbf{T_{ind} = P_{conv} / \omega_m}$$
$$\mathbf{T_{ind} = P_{AG} / \omega_s}$$
$$\mathbf{P_{rcl} = s.P_{AG}}$$
$$\mathbf{P_{conv} = (1-s)\, P_{AG}}$$

- Torque developed by the motor $\mathbf{T_d}$**:**

$$T_d = \frac{P_{\text{gross}}}{\omega_r} = \frac{P_{\text{gross}}}{\omega_s(1-s)} = \frac{3V_1^2 R_2/s}{\omega_s[(R_1 + R_2/s)^2 + (X_1 + X_2)^2]}$$

or $$T_d = \frac{3}{\omega_s} I_2^2 \frac{R_2}{s} \text{ N-m}$$

- Slip at maximum Torque $\mathbf{S_{maxT}}$:

$$S_{\max T} = \pm \frac{R_2}{\sqrt{R_1^2 + (X_1 + X_2)^2}}$$

- Maximum developed torque $\mathbf{T_{max}}$**:**

$$T_{\max} = \frac{3V_{1\text{ph}}^2}{2\omega_s\left[R_1 \pm \sqrt{R_1^2 + (X_1 + X_2)^2}\right]}$$

- Starting orque $\mathbf{T_{st}}$ **:**

$$T_{\text{start}} = \frac{3V_1^2 R_2}{\omega_s\left[(R_1 + R_2)^2 + (X_1 + X_2)^2\right]}$$

UNIT-V

Circle Diagram of Induction Motors & Speed control methods

- **Circle diagram**
- **No load and blocked rotor tests**
- **Predetermination of performance**
- **Methods of starting and starting current and torque calculations.**
- **Speed control: Change of frequency Change of poles and methods of consequent poles**
- **Cascade connection**
- **Injection of an emf into rotor circuit (qualitative treatment only)**
- **Induction generator principle of operation**

Speed control of Induction motors - Basic Methods:

Stator side:

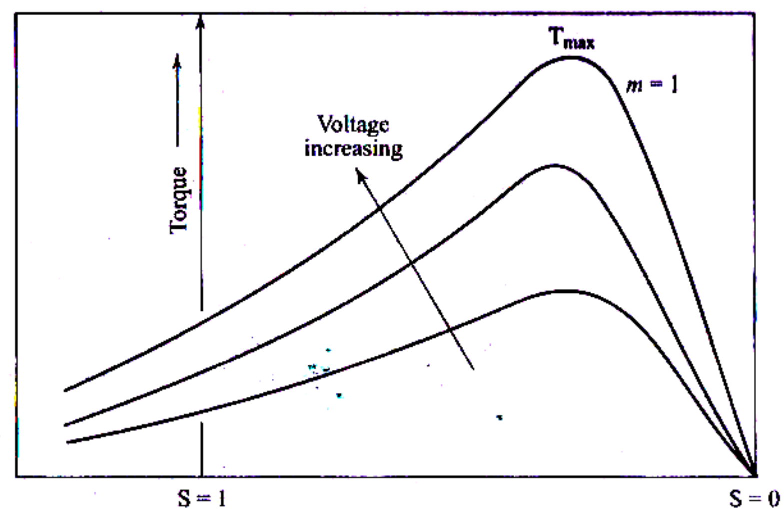
1. Stator Voltage control
2. Stator variable frequency control

Rotor side:

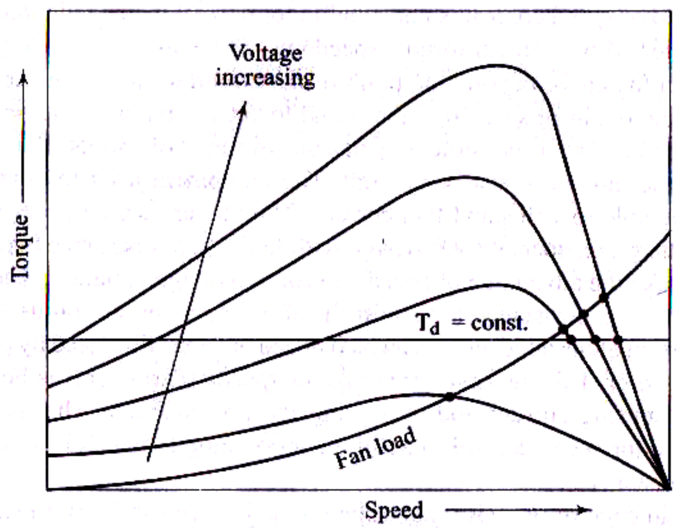
- Rotor resistance control
- Slip-energy recovery

Stator voltage control:

- From the expression for the torque developed by an induction motor, we can see that it is directly proportional to the square of the applied terminal voltage at a constant value of supply frequency and slip. By varying the applied voltage, a set of torque-speed curves as shown below can be obtained. When the applied voltage changes by n times the resulting torque changes by n^2 times.



(a) Typical speed-torque curves for variation in stator voltage (low-resistance rotor)



(b) Operating points and speed range for constant torque and fan type load (rotor resistance low)

- If constant torque is required at different voltages, the slip increases with decreasing voltage to accommodate the required rotor current. But the power factor deteriorates at low voltages.
- Fig(b) shows the torque- speed curves along with a constant load and varying load (with speed).From this it can be seen that speed control is possible only in a limited range

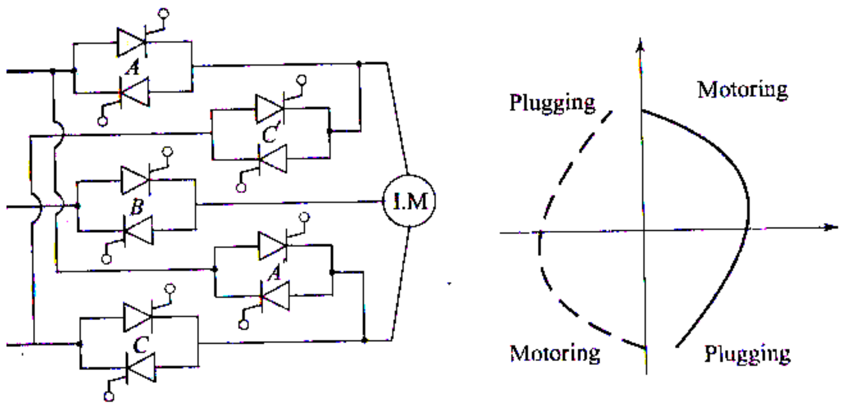
Limitations of Stator voltage control:

- The portion of the speed control beyond the maximum torque is unstable and is not suitable for speed control.
- Normal squirrel cage motors will have low rotor resistance and therefore will have a large unstable region. Hence speed control is possible only in a limited band.
- The starting current is also very high for these motors (because of low rotor resistance). Hence the equipment used for control of these motors must be able to handle/withstand such large starting currents.
- The power factor also will be poor at large slips.
- Therefore special rotor design with high resistance is required to be able to take advantage of speed control with stator voltage variation. This shifts the point of slip for maximum torque to the left and decreases the unstable region.
- The unstable region can be reduced or even completely eliminated by properly designing the rotor. This increases the range of speed control substantially, reduces the starting current and improves the power factor.
- However motors designed with high rotor resistance to achieve higher speed control range will have higher rotor losses at large slips and will have to dissipate the resulting large heat in the Rotor itself.
- But slip ring motors allow the insertion of the high resistance externally .Hence the losses will be dissipated in the external resistors only and Rotor heating will be avoided.

Method of stator voltage control:

AC voltage controllers can be used for varying the applied input stator voltage. By controlling the firing angle of the thyristors connected in anti parallel in each phase the RMS value of the stator voltage applied to each phase can be varied. To get the desired speed control.

Four quadrant operation with plugging is obtained by the use of the circuit shown in the figure below. Thyristor pairs A,B and C provide operation in quadrants 1 &4 (as shown by the solid line) . Thyristor pairs A',B and C' changes the phase sequence and thus provide operation in quadrants 2&3(as shown by the dotted line).



Precaution:

While changing from one set to another set of thyristor pairs, i.e from ABC to A'BC' or *vice versa, care* should be taken to ensure that the incoming pair is activated only after the outgoing pair is fully turned off. This is to avoid short circuiting of the supply by the conducting thyristor pairs. Protection against such faults can be provided only by the fuse links and not by the current control.

Limitations:

A review of the AC controllers reveals that:

- The output voltage from an AC controller is dependent not only on the delay angle of the gate firing pulses but also on the periods of current flow which in turn are dependent on the load power factor. An induction motor will draw a varying power factor current and this will influence the voltage being applied to it. When ever the load current is continuous, the controller will not have any influence on the circuit conditions at all.
- Control is achieved by distortion of the voltage waveforms and by the reduction of the current flow periods. Significant amounts of stator and rotor harmonic currents will flow and eddy currents will be induced in the iron core. These will cause additional motor heating and alter the motor performance compared with sinusoidal operation.

The practical results of these limitations are:

- The motor performance can be predicted only after a full understanding of the motor, thyristor converter and the load.
- A closed loop speed control based on a tachogenerator speed feedback is essential to ensure stable performance.
- The system gains most practical application when the load is predictable and the load torque required at low speeds is relatively low.

As far as the thyristor ratings are concerned:

- The normal crest working voltage is the peak of the supply line voltage, but high transients can occur if the circuit is opened while in operation by switches or fuses.
- The stored energy in the motor has to be allowed for an assessment of thyristor voltage safety margins and surge suppression requirements.
- The most significant factor in current ratings is the possibility of thyristors having to carry the normal motor starting currents during a period when the thyristors are unable to influence the circuit due to adverse load or power factor conditions.

Summary:

Important Concepts and conclusions:

Speed control of Induction motors - Basic Methods:

Stator side:

- Stator Voltage control
- Stator variable frequency control

Rotor side:

- Rotor resistance control
- Slip-energy recovery